

ELSEVIER AEROSPACE ENGINEERING SERIES

Orbital Mechanics for Engineering Students

Third Edition

solution manual

MATLAB
examples

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for Engineering
Students

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SOLUTIONS MANUAL

to accompany

ORBITAL MECHANICS FOR ENGINEERING STUDENTS

Third Edition

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Problem 1.1 Given the three vectors $\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$, $\mathbf{B} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$ and $\mathbf{C} = C_x \hat{\mathbf{i}} + C_y \hat{\mathbf{j}} + C_z \hat{\mathbf{k}}$, show analytically that

- (a) $\mathbf{A} \cdot \mathbf{A} = A^2$
 (b) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$
 (c) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Solution

$$\begin{aligned} \mathbf{A} \cdot \mathbf{A} &= (A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}) (A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}) \\ &= A_x \hat{\mathbf{i}} \cdot (A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}) + A_y \hat{\mathbf{j}} \cdot (A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}) + A_z \hat{\mathbf{k}} \cdot (A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}) \\ &= [A_x^2 (\hat{\mathbf{i}} \cdot \hat{\mathbf{i}}) + A_x A_y (\hat{\mathbf{i}} \cdot \hat{\mathbf{j}}) + A_x A_z (\hat{\mathbf{i}} \cdot \hat{\mathbf{k}})] + [A_y A_x (\hat{\mathbf{j}} \cdot \hat{\mathbf{i}}) + A_y^2 (\hat{\mathbf{j}} \cdot \hat{\mathbf{j}}) + A_y A_z (\hat{\mathbf{j}} \cdot \hat{\mathbf{k}})] \\ &\quad + [A_z A_x (\hat{\mathbf{k}} \cdot \hat{\mathbf{i}}) + A_z A_y (\hat{\mathbf{k}} \cdot \hat{\mathbf{j}}) + A_z^2 (\hat{\mathbf{k}} \cdot \hat{\mathbf{k}})] \\ &= [A_x^2 (1) + A_x A_y (0) + A_x A_z (0)] + [A_y A_x (0) + A_y^2 (1) + A_y A_z (0)] + [A_z A_x (0) + A_z A_y (0) + A_z^2 (1)] \\ &= A_x^2 + A_y^2 + A_z^2 \end{aligned}$$

But, according to the Pythagorean Theorem, $A_x^2 + A_y^2 + A_z^2 = A^2$, where $A = \|\mathbf{A}\|$, the magnitude of the vector \mathbf{A} . Thus $\boxed{\mathbf{A} \cdot \mathbf{A} = A^2}$.

(b)

$$\begin{aligned} \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) &= \mathbf{A} \cdot \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \\ &= (A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}) [\hat{\mathbf{i}}(B_y C_z - B_z C_y) - \hat{\mathbf{j}}(B_x C_z - B_z C_x) + \hat{\mathbf{k}}(B_x C_y - B_y C_x)] \\ &= A_x (B_y C_z - B_z C_y) - A_y (B_x C_z - B_z C_x) + A_z (B_x C_y - B_y C_x) \end{aligned}$$

or

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = A_x B_y C_z + A_y B_z C_x + A_z B_x C_y - A_x B_z C_y - A_y B_x C_z - A_z B_y C_x \tag{1}$$

Note that $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$, and according to (1)

$$\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = C_x A_y B_z + C_y A_z B_x + C_z A_x B_y - C_x A_z B_y - C_y A_x B_z - C_z A_y B_x \tag{2}$$

The right hand sides of (1) and (2) are identical. Hence $\boxed{\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}}$.

(c)

$$\begin{aligned}
 \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= (A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}) \times \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_y C_z - B_z C_y & B_z C_x - B_x C_y & B_x C_y - B_y C_x \end{vmatrix} \\
 &= [A_y (B_x C_y - B_y C_x) - A_z (B_z C_x - B_x C_z)] \hat{\mathbf{i}} + [A_z (B_y C_z - B_z C_y) - A_x (B_x C_y - B_y C_x)] \hat{\mathbf{j}} \\
 &\quad + [A_x (B_z C_x - B_x C_z) - A_y (B_y C_z - B_z C_y)] \hat{\mathbf{k}} \\
 &= (A_y B_x C_y + A_z B_x C_z - A_y B_y C_x - A_z B_z C_x) \hat{\mathbf{i}} + (A_x B_y C_x + A_z B_y C_z - A_x B_x C_y - A_z B_z C_y) \hat{\mathbf{j}} \\
 &\quad + (A_x B_z C_x + A_y B_z C_y - A_x B_x C_z - A_y B_y C_z) \hat{\mathbf{k}} \\
 &= [B_x (A_y C_y + A_z C_z) - C_x (A_y B_y + A_z B_z)] \hat{\mathbf{i}} + [B_y (A_x C_x + A_z C_z) - C_y (A_x B_x + A_z B_z)] \hat{\mathbf{j}} \\
 &\quad + [B_z (A_x C_x + A_y C_y) - C_z (A_x B_x + A_y B_y)] \hat{\mathbf{k}}
 \end{aligned}$$

Add and subtract the underlined terms to get

$$\begin{aligned}
 \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= [B_x (A_y C_y + A_z C_z + \underline{A_x C_x}) - C_x (A_y B_y + A_z B_z + \underline{A_x B_x})] \hat{\mathbf{i}} \\
 &\quad + [B_y (\underline{A_x C_x} + A_z C_z + \underline{A_y C_y}) - C_y (\underline{A_x B_x} + A_z B_z + \underline{A_y B_y})] \hat{\mathbf{j}} \\
 &\quad + [B_z (\underline{A_x C_x} + A_y C_y + \underline{A_z C_z}) - C_z (\underline{A_x B_x} + A_y B_y + \underline{A_z B_z})] \hat{\mathbf{k}} \\
 &= (B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}) (\underline{A_x C_x} + A_y C_y + A_z C_z) - (C_x \hat{\mathbf{i}} + C_y \hat{\mathbf{j}} + C_z \hat{\mathbf{k}}) (\underline{A_x B_x} + A_y B_y + A_z B_z) \\
 &= (B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}) (\mathbf{A} \cdot \mathbf{C}) - (C_x \hat{\mathbf{i}} + C_y \hat{\mathbf{j}} + C_z \hat{\mathbf{k}}) (\mathbf{A} \cdot \mathbf{B})
 \end{aligned}$$

Or,

$$\boxed{\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})}$$

Problem 1.2 Use just the vector identities in Problem 1.1 to show that

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$$

Solution

From Problem 1.1(b)

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = [(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}] \cdot \mathbf{D} \quad (1)$$

But

$$[(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}] \cdot \mathbf{D} = -[\mathbf{C} \times (\mathbf{A} \times \mathbf{B})] \cdot \mathbf{D}$$

Using Problem 1.1(c) on the right yields

$$[(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}] \cdot \mathbf{D} = -[\mathbf{A}(\mathbf{C} \cdot \mathbf{B}) - \mathbf{B}(\mathbf{C} \cdot \mathbf{A})] \cdot \mathbf{D}$$

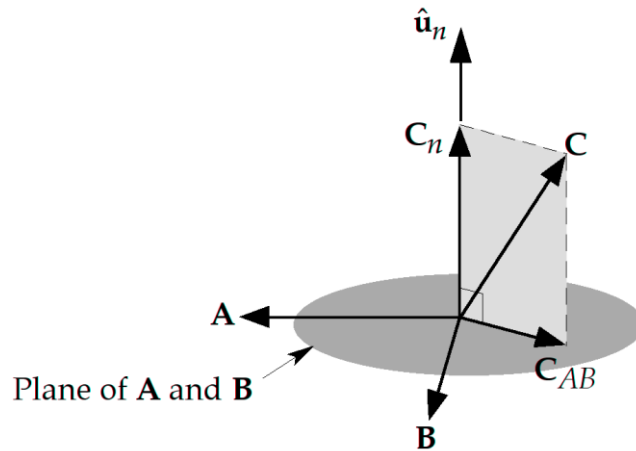
or

$$[(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}] \cdot \mathbf{D} = -(\mathbf{A} \cdot \mathbf{D})(\mathbf{C} \cdot \mathbf{B}) + (\mathbf{B} \cdot \mathbf{D})(\mathbf{C} \cdot \mathbf{A}) \quad (2)$$

Substituting (2) into (1) we get

$$\boxed{(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})}$$

Problem 1.3 Let $\mathbf{A} = 8\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$, $\mathbf{B} = 9\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{C} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$. Calculate the (scalar) projection of C_{AB} of \mathbf{C} onto the plane of \mathbf{A} and \mathbf{B} .



Solution

The unit normal $\hat{\mathbf{u}}_n$ to the plane of \mathbf{A} and \mathbf{B} is

$$\hat{\mathbf{u}}_n = \frac{\mathbf{A} \times \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} = \frac{-63\hat{\mathbf{i}} + 100\hat{\mathbf{j}} - 33\hat{\mathbf{k}}}{17 \cdot 10.863} = -0.34115\hat{\mathbf{i}} + 0.54141\hat{\mathbf{j}} - 0.17870\hat{\mathbf{k}}$$

The projection C_n of \mathbf{C} in the direction of the unit normal to the plane is

$$C_n = \mathbf{C} \cdot \hat{\mathbf{u}}_n = (3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 10\hat{\mathbf{k}}) \cdot (-0.34115\hat{\mathbf{i}} + 0.54141\hat{\mathbf{j}} - 0.17870\hat{\mathbf{k}}) = -0.10289$$

\mathbf{C} is the hypotenuse of the right triangle whose other two sides are of length C_n and C_{AB} , where C_{AB} lies in the plane of \mathbf{A} and \mathbf{B} . Therefore, the square of the length of \mathbf{C} is

$$\|\mathbf{C}\|^2 = C_n^2 + C_{AB}^2$$

That is

$$3^2 + 5^2 + 10^2 = (-0.10289)^2 + C_{AB}^2$$

which means

$$\boxed{C_{AB} = 11.575}$$

Problem 1.4 Since $\hat{\mathbf{u}}_t$ and $\hat{\mathbf{u}}_n$ are perpendicular and $\hat{\mathbf{u}}_t \times \hat{\mathbf{u}}_n = \hat{\mathbf{u}}_b$, use the *bac-cab* rule to show that $\hat{\mathbf{u}}_b \times \hat{\mathbf{u}}_t = \hat{\mathbf{u}}_n$ and $\hat{\mathbf{u}}_n \times \hat{\mathbf{u}}_b = \hat{\mathbf{u}}_t$, thereby verifying Equation 1.29.

Solution

bac-cab rule: $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

$$\begin{aligned}\hat{\mathbf{u}}_b \times \hat{\mathbf{u}}_t &= (\hat{\mathbf{u}}_t \times \hat{\mathbf{u}}_n) \times \hat{\mathbf{u}}_t \\ &= -\hat{\mathbf{u}}_t \times (\hat{\mathbf{u}}_t \times \hat{\mathbf{u}}_n) \\ &= -[\hat{\mathbf{u}}_t (\hat{\mathbf{u}}_t \cdot \hat{\mathbf{u}}_n) - \hat{\mathbf{u}}_n (\hat{\mathbf{u}}_t \cdot \hat{\mathbf{u}}_t)] \\ &= -[\hat{\mathbf{u}}_t (0) - \hat{\mathbf{u}}_n (1)]\end{aligned}$$

$$\therefore \boxed{\hat{\mathbf{u}}_b \times \hat{\mathbf{u}}_t = \hat{\mathbf{u}}_n}$$

$$\begin{aligned}\hat{\mathbf{u}}_n \times \hat{\mathbf{u}}_b &= \hat{\mathbf{u}}_n \times (\hat{\mathbf{u}}_t \times \hat{\mathbf{u}}_n) \\ &= \hat{\mathbf{u}}_t (\hat{\mathbf{u}}_n \cdot \hat{\mathbf{u}}_n) - \hat{\mathbf{u}}_n (\hat{\mathbf{u}}_n \cdot \hat{\mathbf{u}}_t) \\ &= \hat{\mathbf{u}}_t (1) - \hat{\mathbf{u}}_n (0)\end{aligned}$$

$$\therefore \boxed{\hat{\mathbf{u}}_n \times \hat{\mathbf{u}}_b = \hat{\mathbf{u}}_t}$$

Problem 1.5 The x , y and z coordinates (in meters) of a particle as a function of time (in seconds) are $x = \sin 3t$, $y = \cos t$ and $z = \sin 2t$. At $t = 3\text{ s}$ determine:

- The velocity \mathbf{v} , in Cartesian coordinates.
- The speed v .
- The unit tangent $\hat{\mathbf{u}}_t$.
- The angles θ_x , θ_y and θ_z that \mathbf{v} makes with the x , y and z axes.
- The acceleration \mathbf{a} in Cartesian coordinates.
- The unit binormal vector $\hat{\mathbf{u}}_b$.
- The unit normal vector $\hat{\mathbf{u}}_n$.
- The angles ϕ_x , ϕ_y and ϕ_z that \mathbf{a} makes with the x , y and z axes.
- The tangential component a_t of the acceleration.
- The normal component a_n of the acceleration.
- The radius of curvature of the path of P .
- The Cartesian coordinates of the center of curvature of the path.

Solution

(a)

$$\mathbf{v} = \left. \frac{d\mathbf{r}}{dt} \right|_{t=3} = \left. \frac{d}{dt} (\sin 3t\hat{\mathbf{i}} + \cos t\hat{\mathbf{j}} + \sin 2t\hat{\mathbf{k}}) \right|_{t=3} = 3\cos 3t\hat{\mathbf{i}} - \sin t\hat{\mathbf{j}} + 2\cos 2t\hat{\mathbf{k}} \Big|_{t=3}$$

$$\boxed{\mathbf{v} = -2.2791\hat{\mathbf{i}} + 0.95892\hat{\mathbf{j}} - 1.6781\hat{\mathbf{k}} \text{ (m/s)}}$$

(b)

$$v = \|\mathbf{v}\| = \sqrt{(-2.2791)^2 + 0.95892^2 + (-1.6781)^2}$$

$$\boxed{v = 2.9883 \text{ m/s}}$$

(c)

$$\hat{\mathbf{u}}_t = \frac{\mathbf{v}}{v} = \frac{-2.2791\hat{\mathbf{i}} + 0.95892\hat{\mathbf{j}} - 1.6781\hat{\mathbf{k}}}{2.9883}$$

$$\boxed{\hat{\mathbf{u}}_t = -0.76267\hat{\mathbf{i}} + 0.32089\hat{\mathbf{j}} - 0.56157\hat{\mathbf{k}}}$$

(d)

$$\theta_x = \cos^{-1}(\hat{\mathbf{u}}_t \cdot \hat{\mathbf{g}}_x) = \cos^{-1}(-0.76267)$$

$$\boxed{\theta_x = 139.70^\circ}$$

$$\theta_y = \cos^{-1}(\hat{\mathbf{u}}_t \cdot \hat{\mathbf{g}}_y) = \cos^{-1}(0.32089)$$

$$\boxed{\theta_y = 71.283^\circ}$$

$$\theta_z = \cos^{-1}(\hat{\mathbf{u}}_t \cdot \hat{\mathbf{g}}_z) = \cos^{-1}(-0.56157)$$

$$\boxed{\theta_z = 124.16^\circ}$$

(e)

$$\mathbf{a} = \left. \frac{d\mathbf{v}}{dt} \right|_{t=3} = \left. \frac{d}{dt} (3\cos 3t\hat{\mathbf{i}} - \sin t\hat{\mathbf{j}} + 2\cos 2t\hat{\mathbf{k}}) \right|_{t=3} = -9\sin 3t\hat{\mathbf{i}} - \cos t\hat{\mathbf{j}} - 4\sin 2t\hat{\mathbf{k}} \Big|_{t=3}$$

$$\mathbf{a} = -5.8526\hat{\mathbf{i}} - 0.28366\hat{\mathbf{j}} + 2.1761\hat{\mathbf{k}} \quad (\text{m/s}^2)$$

(f)

$$\hat{\mathbf{u}}_b = \frac{\mathbf{v} \times \mathbf{a}}{\|\mathbf{v} \times \mathbf{a}\|} = \frac{(-2.2791\hat{\mathbf{i}} + 0.95892\hat{\mathbf{j}} - 1.6781\hat{\mathbf{k}}) \times (-5.8526\hat{\mathbf{i}} - 0.28366\hat{\mathbf{j}} + 2.1761\hat{\mathbf{k}})}{\|(-2.2791\hat{\mathbf{i}} + 0.95892\hat{\mathbf{j}} - 1.6781\hat{\mathbf{k}}) \times (-5.8526\hat{\mathbf{i}} - 0.28366\hat{\mathbf{j}} + 2.1761\hat{\mathbf{k}})\|}$$

$$= \frac{1.6107\hat{\mathbf{i}} + 14.781\hat{\mathbf{j}} + 6.2587\hat{\mathbf{k}}}{\|1.6107\hat{\mathbf{i}} + 14.781\hat{\mathbf{j}} + 6.2587\hat{\mathbf{k}}\|} = \frac{1.6107\hat{\mathbf{i}} + 14.781\hat{\mathbf{j}} + 6.2587\hat{\mathbf{k}}}{16.132}$$

$$\hat{\mathbf{u}}_b = 0.099844\hat{\mathbf{i}} + 0.91625\hat{\mathbf{j}} + 0.38797\hat{\mathbf{k}}$$

(g)

$$\hat{\mathbf{u}}_n = \hat{\mathbf{u}}_b \times \hat{\mathbf{u}}_t = (0.099844\hat{\mathbf{i}} + 0.91625\hat{\mathbf{j}} + 0.38797\hat{\mathbf{k}}) \times (-0.76267\hat{\mathbf{i}} + 0.32089\hat{\mathbf{j}} - 0.56157\hat{\mathbf{k}})$$

$$\hat{\mathbf{u}}_n = -0.63904\hat{\mathbf{i}} - 0.23982\hat{\mathbf{j}} + 0.73083\hat{\mathbf{k}}$$

(h)

$$a = \|\mathbf{a}\| = \sqrt{(-5.8526)^2 + (-0.28366)^2 + (2.1761)^2} = 6.2505 \text{ m/s}^2$$

$$\phi_x = \cos^{-1}\left(\frac{a_x}{a}\right) = \cos^{-1}\left(\frac{-5.8526}{6.2505}\right)$$

$$\phi_x = 159.45^\circ$$

$$\phi_y = \cos^{-1}\left(\frac{a_y}{a}\right) = \cos^{-1}\left(\frac{-0.28366}{6.2505}\right)$$

$$\phi_y = 92.601^\circ$$

$$\phi_z = \cos^{-1}\left(\frac{a_z}{a}\right) = \cos^{-1}\left(\frac{2.1761}{6.2505}\right)$$

$$\phi_z = 69.626^\circ$$

(i)

$$a_t = \mathbf{a} \cdot \hat{\mathbf{u}}_t = (-5.8526\hat{\mathbf{i}} - 0.28366\hat{\mathbf{j}} + 2.1761\hat{\mathbf{k}}) \cdot (-0.76267\hat{\mathbf{i}} + 0.32089\hat{\mathbf{j}} - 0.56157\hat{\mathbf{k}})$$

$$a_t = 3.1505 \text{ m/s}^2$$

(j)

$$a_n = \mathbf{a} \cdot \hat{\mathbf{u}}_n = (-5.8526\hat{\mathbf{i}} - 0.28366\hat{\mathbf{j}} + 2.1761\hat{\mathbf{k}}) \cdot (-0.63904\hat{\mathbf{i}} - 0.23982\hat{\mathbf{j}} + 0.73083\hat{\mathbf{k}})$$

$$\boxed{a_n = 5.3984 \text{ m/s}^2}$$

(k)

$$\rho = \frac{v^2}{a_n} = \frac{2.9883^2}{5.3984}$$

$$\boxed{\rho = 1.6542 \text{ m}}$$

(l)

$$\mathbf{r}_C = \mathbf{r} + \rho \hat{\mathbf{u}}_n$$

$$= (0.65029\hat{\mathbf{i}} + 0.28366\hat{\mathbf{j}} - 0.54402\hat{\mathbf{k}}) + 1.6542(-0.63904\hat{\mathbf{i}} - 0.23982\hat{\mathbf{j}} + 0.73083\hat{\mathbf{k}})$$

$$\boxed{\mathbf{r}_C = -0.40678\hat{\mathbf{i}} - 0.11304\hat{\mathbf{j}} + 0.66489\hat{\mathbf{k}} \text{ (m)}}$$

Problem 1.6 An 80 kg man and 50 kg woman stand 0.5 meter from each other. What is the force of gravitational attraction between the couple?

Solution

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.6742 \times 10^{-11} \cdot 80 \cdot 50}{0.5^2}$$

$$F = 1.0679 \times 10^{-6} \text{ N}$$

Problem 1.7 If a person's weight is W on the surface of the earth, calculate the earth's gravitational pull on that person at a distance equal to the moon's orbit.

Solution

Let m be the person's mass, M_E the mass of the earth, and R_E the radius of the earth. Then

$$W = \frac{GmM_E}{R_E^2}$$

or

$$GmM_E = WR_E^2 \quad (1)$$

If r_{moon} is the moon's orbital radius, then the gravitational pull of the earth at that distance is

$$F = \frac{GmM_E}{r_{\text{moon}}^2} \quad (2)$$

Substituting (1) into (2) yields

$$F = \left(\frac{R_E}{r_{\text{moon}}} \right)^2 W$$

Since $R_E = 6378 \text{ km}$ and $r_{\text{moon}} = 384\,400 \text{ km}$, we find

$$\boxed{F = 0.0002753W}$$

Problem 1.8 If a person's weight is W on the surface of the earth, calculate what it would be, in terms of W , at the surface of

- (a) the moon
- (b) Mars
- (c) Jupiter

Solution

The force of gravity on mass m at the earth's surface is

$$W = \frac{GmM_E}{R_E^2}$$

so that

$$Gm = \frac{WR_E^2}{M_E} \tag{1}$$

At the surface of planet P , the force of gravity is. using (1),

$$W_P = \frac{GmM_P}{R_P^2} = \left(\frac{R_E}{R_P}\right)^2 \frac{M_P}{M_E} W \tag{2}$$

(a)

$$W_{\text{moon}} = \left(\frac{R_E}{R_{\text{moon}}}\right)^2 \frac{M_{\text{moon}}}{M_E} W = \left(\frac{6378}{1737}\right)^2 \frac{7.348 \times 10^{22}}{597.4 \times 10^{22}} W$$

$$\boxed{W_{\text{moon}} = 0.1658W}$$

(b)

$$W_{\text{Mars}} = \left(\frac{R_E}{R_{\text{Mars}}}\right)^2 \frac{M_{\text{Mars}}}{M_E} W = \left(\frac{6378}{3396}\right)^2 \frac{64.19 \times 10^{22}}{597.4 \times 10^{22}} W$$

$$\boxed{W_{\text{Mars}} = 0.3790W}$$

(c)

$$W_{\text{Jupiter}} = \left(\frac{R_E}{R_{\text{Jupiter}}}\right)^2 \frac{M_{\text{Jupiter}}}{M_E} W = \left(\frac{6378}{71490}\right)^2 \frac{189900 \times 10^{22}}{597.4 \times 10^{22}} W$$

$$\boxed{W_{\text{Jupiter}} = 2.530W}$$

Problem 1.9 A satellite of mass m is in a circular orbit around the earth, whose mass is M . The orbital radius from the center of the earth is r . Use Newton's Second Law of motion, together with Equations 1.25 and 1.40, to calculate the speed v of the satellite in terms of M , r and the gravitational constant G .

Solution

Writing Newton's Second Law in the direction normal to the circular orbital path,

$$F_n = ma_n \tag{1}$$

From Equation 1.25

$$a_n = \frac{v^2}{r}$$

From Equation 1.40m

$$F_n = \frac{GmM}{r^2}$$

Therefore, (1) becomes

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

so that

$$v = \sqrt{\frac{GM}{r}}$$