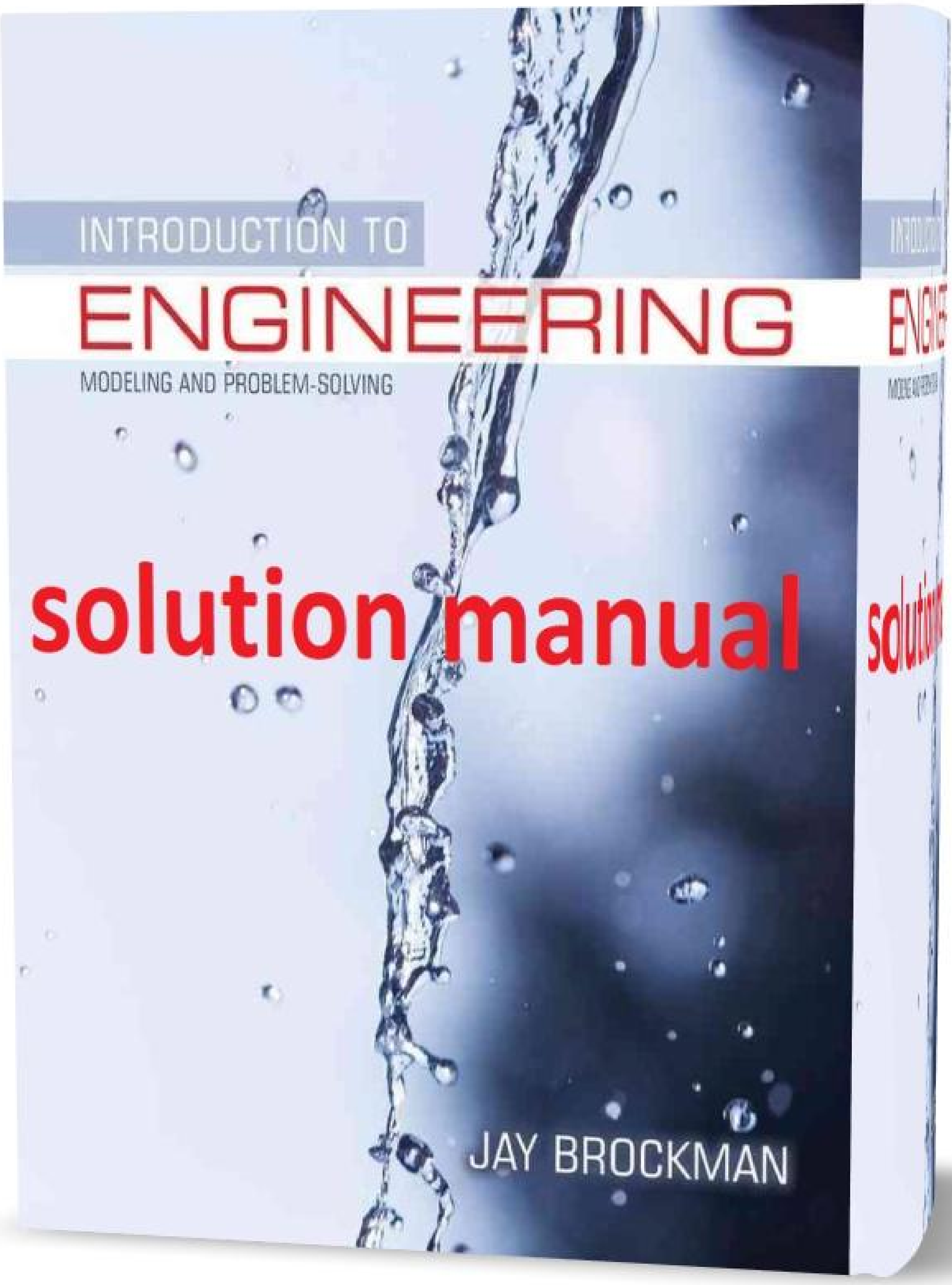

INTRODUCTION TO
ENGINEERING
MODELING AND PROBLEM-SOLVING
solution manual
JAY BROCKMAN

Chapter 3

Chapter 3

Problem 1: Concept Map: Expertise and the Learning Process

No solutions posted for this problem.

Problem 2: Concept Map: Engineering Disciplines

No solutions posted for this problem.

Problem 3: Studying with Music

Solution 3.1: Shannon's Solution

Yes.

Problem 4: Levels of Understanding: Making Copies

Solution 4.1: Jay's Solution

1. Mike gets a job working as a copyboy. Today is his first day, and within the first hour, the copier runs out of paper. Mike refills the copier.

- There may have been a small amount of *analysis* required to determine that the copier was out of paper, but more likely it was simply *comprehending* and error message on the control panel of the copier, and then *applying* a well-known procedure.

2. Mike needs to make copies of 10 articles for one of his business' employees.

- Suppose that Mike was simply told to make copies of the 10 articles. He would *analyze* this problem and then *synthesize*
a plan for doing this in the most efficient way. For example, if several articles were next to each other in the

proceed differently.

3. The papers jam in the middle of making the copies. Mike needs to fix the copier and finish making the copies. Luckily, the screen on the copier gives explicit directions to remove the paper jam.

- Since the screen gives explicit directions, this most likely just involves *comprehension* of the message and then *application* of a well-known procedure.

4. Mike is having a very trying day. When he tries to remove the paper jam, he accidentally changes some settings on the copier and the copies are not the same as the original ones. Mike rectifies this situation.

- Here, Mike had to *analyze* the situation to determine what the problem was.

5. A message shows up on the copier panel telling Mike that the toner is low. He must fix this problem as well.

- This just involves *comprehending* the message and *applying* the procedure to fix it.

6. Finally, Mike's day ends. He is upset with the issues with the copier, and he is contemplating getting a new job that he enjoys more. In the process of finding a job, he does a lot of research and eventually finds the perfect job: as a go-kart tester.

- A classic case of Mike *evaluating* his options.

Problem 5: Levels of Understanding: Making Up Your Own Questions

No solutions posted for this problem.

Problem 6: Hmm . . .

Solution 6.1: Jay's Solution

This problems test *application* of Bloom's Taxonomy.

Problem 7: Practice with the Problem-Solving Framework

Solution 7.1: Jay's Solution

Part 1:

In one long-distance phone call, Amy talked to her parents for twice as long as her brother talked. Her sister talked for 12 minutes longer than Amy. If the phone call was 62 minutes long, how long did each person talk on the phone?

Given:
A set of 3 relationships referring to how long Amy, her brother and her sister talked on a single phone call to their parents.

Find: How long each person spoke

Plan

1. Define variables for how long each person spoke

2. Express the 3 relationships in the word problem in terms of these 3 variables
3. Solve the 3 equations in 3 variables using substitution

Analysis:

- Variables for minutes on phone: Amy (A), brother (B), sister (S)
- Express relationships and solve

Sub (1) into (2)

(4) $S = 12 + 2B$

Sub (1) & (4) into (3)

$$2B + B + (12 + 2B) = 62$$
$$5B + 12 = 62$$
$$5B = 50$$
$$\boxed{B = 10}$$

(1) → $\boxed{A = 2B = 20}$

(2) → $\boxed{S = 12 + A = 32}$

Thus Amy was on the phone for 20 min, her brother for 10, and her sister for 32.

Comments:

* The main heuristic employed was **restating in simpler terms**, rewriting the word problem as an equation.

Part 2:

Roberto needs to draw a line that is 5 inches long, but he does not have a ruler. he does have some sheets of notebook paper that are each 1.5 inches wide and 11 inches long. Describe how Roberto can use the notebook paper to measure 6 inches.

Given: strips of paper 1.5 x 11.0 inches

Find: a process to measure lengths of either 5 or 6 inches

Plan:

- use the fact that we can make measurements in increments of 1.5 inches from the width of the paper strips
- use the fact that we can subtract known lengths from 11 to get other lengths
- note that 5 + 6 = 11

Analysis:

1. draw a line 6 inches long using 4 widths of paper (1.5 X 4 = 6)

2. subtract a measurement 6 inches long from the length of paper to get 5 inches (11 - 6 = 5)

Comments:

Solving this problem used a combination of **working forward** by starting with a strip of paper and seeing that we could use it to measure out 6 inches, and then **working backwards** to get the 5 inches from the 6 inch measurement and the fact that the paper is 11 inches long. There was also implicit **solving a simpler problem** from the hint that was given asking how to find a line 6 inches long. I think the problem would have been considerably tougher if this hint weren't given!

Part 3:

As the hands of a clock move from 6:00 AM to 6:00 PM, how many times to the hands form a right angle?

Given: A clock with hour and minute hands

Find: How many times the hands form a right angle between 6 AM and 6 PM

Plan:

- Try a few test cases
- Look for a pattern
- Check for exceptions

Analysis:

In general, the hands will form a right angle whenever the minute hand is pointing at a number approximately 3 units ahead (clockwise) or 9 units ahead of the hour hand. In most cases, the minute hand won't be pointing exactly at a digit because the hour hand moves. For example, between 12:00 and 1:00, the two times that we get a 90 degree angle between the hands is approximately 12:16 and 12:49

Thus in *most*
cases, there will be two times per hour when the hands form a right angle. There's one exception, however. Between 8:00 and 9:00, the first time that the clocks form a right angle is approximately 8:26, but the second time *is 9:00*, which is of course also the first time that the hands form a right angle between 9:00 and 10:00. Thus there are only 3 times that they form a right angle between 8:00 and 10:00, rather than the usual 4.

The same thing happens at 3:00. Thus over the 12 hour period, there are 12*2 - 2 = 22 times that the hands form a right angle.

Comments:

The heuristics that I used in solving this problem included:

- Draw a picture/Use models--I needed to look at an actual clock to solve this; couldn't do it in my head!
- Divide and conquer--I broke the problem down into two main cases: what happens within a typical hour, and are there any exceptions.
- Solved a related problem?--I noticed the exception at 9:00 first, and from this determined that there would also be an exception at 3:00.

Part 4:

Alice, Nathan, and Marie play in the school band. One plays the drum, one plays the saxophone, and one plays the flute. Alice is a senior. Alice and the saxophone player practice together after school. Nathan and the flute player are sophomores. Who plays which instrument?

Given: a set of clues regarding Alice, Nathan, and Marie, and the instruments drum, sax, and flute.

*Find: who plays which instrument

Plan:

- Draw a table with people labeling each column and instruments labeling each row.
- Using the table to keep track, use the clues to establish who is or is not playing which instrument. Sort of like playing Sudoku!

Analysis:

We'll start with a blank table, where a given cell notes whether or not we know if the person in the column heading plays the instrument in the row heading. A blank means we don't know, a T means "true" and an F means "false"

	Alice	Nathan	Marie
Drum			
Sax			
Flute			

From the clue, "Alice and the sax player practice together," we know that Alice doesn't play sax:

	Alice	Nathan	Marie
Drum			
Sax	F		

Flute			

From the clues, and "Nathan and the flute player are sophomores," we know that Nathan doesn't play flute, and from "Alice is a senior" Alice doesn't play flute, either, so she must play drum.

	Alice	Nathan	Marie
Drum	T		
Sax	F		
Flute	F	F	

Assuming that a person only plays one instrument in the band, we now know enough to fill in the rest of the table:

	Alice	Nathan	Marie
Drum	T	F	F
Sax	F	T	F
Flute	F	F	T

Thus Alice plays drum, Nathan plays sax, and Marie plays flute.

Comments:

Heuristics used in solving this problem were:

- Draw a picture--using the table
- Divide and conquer--solve for Alice first and then use this to help conquer the rest of the problem
- Working forward and backward--in the application of clues

Problem 8: Assumptions and Approximations in Solving Energy Problems

Solution 8.1: Jay's Solution to Part 3 (Reese's Cups)

Part 1:

Part 2:

Part 3:

Given: My favorite candy (Reese's Cups) and exercise through walking

Find: How far you would have to walk to burn the calories contained in the candy

Plan:

- Look up how many calories in a serving of Reese's Cup
- Look up how many calories burned by walking a given distance

Analysis:

According to Hershey's web site (http://www.hersheys.com/products/details/reesespeanutbuttercups.asp), a single serving of Reese's Cups has 260 calories.

Using the chart for calories burned per mile to About.com (http://walking.about.com/cs/howtoloseweight/a/howcalburn.htm), I'll estimate approximately 85 calories per mile. Thus to burn 260 calories would require walking 260/85 = approximately 3 miles.

Comment:

Given that the closest store that sells Resse's Cups is approximately 3 miles from my house, this works out pretty well!

Problem 9: Storing Text on a DVD

Solution 9.1: Jay's Solution

Given: that a letter requires 1 byte of data and DVDs for storage

Find: how many DVDs it would take to store all of the text in books in your school's library

Plan:

- Look up how many bytes per DVD
- Estimate how many bytes of text in all the books in your library
  - How many books in the library?
  - How many pages per book?
  - How many letters per page?
    - How many letters per word?
    - How many words per line?
    - How many lines per page?
- Solve

Analysis:

According to the DVD Forum FAQ (http://www.dvdforum.org/faq-dvdprimer.htm located via a Google Search on "DVD capacity"), the most common DVD format (single-layer, single-sided) has a storage capacity of 4.7 GB. There are other formats available, but we'll assume this one for our calculations.

According to its web site, the University of Notre Dame Library has approximately 2 million books, not counting microfiche and other media.

After looking at a couple of textbooks and novels, I came up with the following estimates of the number of letters per book (including numbers, spaces, and punctuation).

$$500\,\frac{\text{page}}{\text{book}} \times 30\,\frac{\text{line}}{\text{page}} \times 12\,\frac{\text{word}}{\text{line}} \times 500\,\frac{\text{letter}}{\text{word}} = 900,000\,\frac{\text{letter}}{\text{book}}$$

Rounding this up to 1 million, it would thus take approximately 1 MB to store the text of a typical library book in ASCII format.

A 4.7 GB DVD could thus store approximately 4700 books in ASCII format

Storing 2 million books would thus require 2,000,000/4,700 = **425 DVDs**

Comments:

1 MB per book seems like a reasonable number for ASCII text only. For reference, a PDF file of the full *Introduction to Engineering* textbook including all figures, formatting, fonts, etc, is approximately 10 MB.

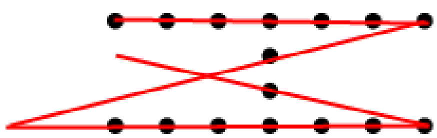
Problem 10: Problem Solving Strategies and Heuristics

Solution 10.1: Jay's Solution

1. Each of the 10 players participates in 9 handshakes, so there are 90 handshake "experiences". But since there are 2 "handshake experiences" for each handshake, there are half as many handshakes as "handshake experiences". Thus there are 45 handshakes.

The heuristic that I used was to first **solve a simpler problem**, namely, thinking about how many handshakes each player experienced. Then I extended this solution to solving the complete problem.

2. Here's a solution:



The heuristic I employed here was **checking for unnecessary constraints**, as one might be tempted to think that there is a constraint to stay within the bounding box of the dots.

Problem 11: Proofs Come in Handy: The Mutilated Chessboard

Solution 11.1: Jay's Solution

1. First, suppose that you have a collection of dominoes that are each exactly the same size as two squares on a chessboard. Is it possible to cover all of the squares on a standard, non-mutilated board with exactly 32 dominoes? How do you know?

- Sure, it's possible. Simply arrange 4 dominoes over each of the 8 rows. The way that we know--and the easiest way to convince someone that you have a solution--is through a demonstration.

2. Now, is it possible to cover all of the squares in a mutilated chessboard with exactly 31 dominoes? How do you know? If it turns out that it’s not possible, how can you prove this? Try to solve this problem without looking up the answer, even though it’s readily available online.

- It turns out that it's not possible. Here's a proof:
  - Every domino placed on the board will cover 2 squares, one black and one white, regardless of whether it it placed vertically or horizontally.
  - In the mutilated chessboard, when you remove two opposite corners, you are removing two squares of the same color. Thus the number of black squares and white squares are unequal (30 of one and 32 of the other).
  - It's impossible to place 31 dominoes in a manner that would cover unequal numbers of black and white squares, so it is not possible to cover the mutilated chessboard with dominoes.

3. Explain how this problems illustrates both strengths and limitations of of physical demonstrations in solving problems.

- The strategy that we used to determine that there was a solution for the non-mutilated chessboard--providing a physical demonstration--would utterly fail in the case of the mutilated chessboard since there is no solution. Further, we can't really use the fact that we can't easily find a physical demonstration of covering the mutilated board to *prove* that there is no solution, because we would have to try *all possible combinations* for this to suffice as a proof. The logical approach that we used is clearly the better way of demonstrating that no solution exists.

Problem 12: Guess and Check to Determine an Average

Solution 12.1: Converging to an average

We want to find the average of the numbers by using the guess and check method described in the book, and without using division. This method will allow us to converge to a solution.

The first step is to make an educated guess of the average, based on the numbers given. Here, the numbers are: 16, 28, 12, 17, 23, 21. So 20 seems to be a good easy guess to start with. If all six numbers were equal to 20, the numbers' sum would be 120 (20x6). The sum of our original six numbers however is 117. So our guess of 20 is high, and we need to go lower (note that we cannot take the difference, 3, and divide that by 6 to get our average since division is not allowed). So our next guess, based on the feedback from above, can be 19. 19x6 is 114, so we're low, and we try 19.5, which happens to be the correct average. For other examples, you would continue until you reached a point where the error is below your tolerable error level.

This method is of course similar to the binary search.

Problem 13: Rabbit and Turtles

Solution 13.1: Time perspective

Observation:

The key observation for this problem is that rather than track all of the small distances the rabbit travels, we can solve the problem by figuring out how long it takes for the turtles to meet. Once we know this time, we can easily calculate the distance the rabbit travels. This is because the amount of time it takes the turtles to meet each other is the same amount of time that the rabbit travels.

Calculations:

Time for turtles to meet each other:

$300m$ = time travelled $\times$ speed of turtle A + time travelled $\times$ speed of turtle B

$300m$ = time travelled $\times$ 80m/h + time travelled $\times$ 70m/h

$300m$ = time travelled $\times$ 150m/h

$300m/150m/h$ = time travelled

time travelled = $2h$

Distance rabbit travels:

distance = $137m/h$ * time travelled

distance = $137m/h$ * $2h$

distance = $274m$

Problem 14: Does Multiplication Equal Addition?

Solution 14.1: Infinite is not enough!

This problem illustrates than even if you have an infinite number of examples to prove a hypothesis, it still does not make it necessarily true.

In our example, we want to have the following equality, given two numbers a and b: a x b = a + b

Therefore, given a number a, we solve for b: b = a / (a - 1)

That's how the pairs (2, 2), (3, 1.5), etc. are all pairs where their sum is equal to their product. There are therefore an infinite number of such pairs. Concluding that adding and multiplying are equivalent would obviously be a classic example of sophistry.

Another example, actually much simpler: the claim that "all integers are odd" will have an infinite number of examples to try to prove it, but is nonetheless a false statement.

We hope that this problem will help students recognize that a tight view of a system, without seeing the bigger picture, and without accepting counter-examples nor taking in other views, will lead to disastrous results.

Problem 15: What Do These Problems Have in Common?

Solution 15.1: Jay's Solution

Each of these problems require an *application* level of understanding of basic physical laws.

Problem 16: Graphical Insights on Quadratic Equations

Solution 16.1: Jay's Solution

Using either factoring or the quadratic formula, we find that each of the given polynomials have the real roots listed in the table below, as well as the number of crossings of the x-axis as shown in plots of the equations.

polynomial	real roots	x-axis crossings
$x^2 - 5x + 6$	2,3	2
$x^2 - 2x + 1$	1 (repeated)	1
$x^2 - 3x + 4$	no real roots	0