Chapter 2

Exercise Solutions

Ex. 2.1

(a) $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{\left(6.625\times10^{-34}\right)\left(3\times10^{10}\right)}{100\times10^{-8}}$

$= 1.9875\times10^{-17}\ \text{J}$

or $E = \dfrac{1.9875\times10^{-17}}{1.6\times10^{-19}} = 124\ \text{eV}$

(b) $E = \dfrac{hc}{\lambda} = \dfrac{\left(6.625\times10^{-34}\right)\left(3\times10^{10}\right)}{4500\times10^{-8}}$

$= 4.417\times10^{-19}\ \text{J}$

or $E = \dfrac{4.417\times10^{-19}}{1.6\times10^{-19}} = 2.76\ \text{eV}$

Ex 2.2

(a) $p = \sqrt{2mE}$

$= \left[2\left(9.11\times10^{-31}\right)\left(12\times10^{-3}\right)\left(1.6\times10^{-19}\right)\right]^{1/2}$

$= 5.915\times10^{-26}\ \text{kg-m/s}$

$\lambda = \dfrac{h}{p} = \dfrac{6.625\times10^{-34}}{5.915\times10^{-26}} = 1.12\times10^{-8}\ \text{m}$

or $\lambda = 112\ \overset{o}{A}$

(c) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{112\times10^{-10}}$

$= 5.915\times10^{-26}\ \text{kg-m/s}$

$E = \dfrac{1}{2}\dfrac{p^2}{m} = \dfrac{1}{2}\dfrac{\left(5.915\times10^{-26}\right)^2}{2.2\times10^{-31}}$

$= 7.952\times10^{-21}\ \text{J}$

or $E = \dfrac{7.952\times10^{-21}}{1.6\times10^{-19}} = 4.97\times10^{-2}\ \text{eV}$

Ex 2.3

(a) $E_n = \dfrac{\hbar^2\pi^2n^2}{2ma^2}$

$= \dfrac{\left(1.054\times10^{-34}\right)^2\pi^2n^2}{2\ \left(9.11\times10^{-31}\right)\left(12\times10^{-10}\right)^2}$

$= 4.179\times10^{-20}n^2\ \text{J}$

or $E_n = \dfrac{4.179\times10^{-20}n^2}{1.6\times10^{-19}} = 0.261n^2\ \text{eV}$

Then

$E_1 = 0.261\text{eV},\ E_2 = 1.045\text{eV},\ E_3 = 2.351\text{eV}$

(b) $E_n = \dfrac{\hbar^2\pi^2n^2}{2ma^2}$

$= \dfrac{\left(1.054\times10^{-34}\right)^2\pi^2n^2}{2\left(1.67\times10^{-27}\right)\left(12\times10^{-10}\right)^2}$

$= 2.28\times10^{-23}n^2\ \text{J}$

or $E_n = \dfrac{2.27967\times10^{-23}n^2}{1.6\times10^{-19}}$

$= 1.425\times10^{-4}n^2\ \text{eV}$

Then $E_1 = 1.425\times10^{-4}\ \text{eV}$

$E_2 = 5.70\times10^{-4}\ \text{eV}$

$E_3 = 1.28\times10^{-3}\ \text{eV}$

Ex 2.4

$E = \dfrac{1}{2}m\upsilon^2 = \dfrac{1}{2}\left(9.11\times10^{-31}\right)\left(10^5\right)^2$

$= 4.555\times10^{-21}\ \text{J}$

Now

$k_2 = \sqrt{\dfrac{2m}{\hbar^2}\left(V_o - E\right)}$ Set $V_o = 3E$

Then

$k_2 = \dfrac{1}{\hbar}\sqrt{2m\left(2E\right)}$

$= \dfrac{\left[2\left(9.11\times10^{-31}\right)\left(2\right)\left(4.555\times10^{-21}\right)\right]^{1/2}}{1.054\times10^{-34}}$

or

$k_2 = 1.222\times10^{9}\ \text{m}^{-1}$

$P = \exp\left[-2k_2d\right]$

(a) $d = 10\ \overset{o}{A} = 10\times10^{-10}\ \text{m}$

$P = \exp\left[-\left(2\right)\left(1.222\times10^{9}\right)\left(10\times10^{-10}\right)\right]$

or

$P = 0.0868 \Rightarrow 8.68\,\%$

(b) $d = 100\ \overset{o}{A} = 100\times10^{-10}\ \text{m}$

$P = \exp\left[-\left(2\right)\left(1.222\times10^{9}\right)\left(100\times10^{-10}\right)\right]$

or

$P = 2.43\times10^{-11} \Rightarrow 2.43\times10^{-9}\,\%$

Ex 2.5

(a) $$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$
$$= \sqrt{\frac{2(9.11\times10^{-31})(1.2-0.12)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}}$$
$$= 5.3236\times10^9 \text{ m}^{-1}$$
Then
$$T \cong 16\left(\frac{0.12}{1.2}\right)\left(1-\frac{0.12}{1.2}\right) \times \exp\left[-2(5.3236\times10^9)(5\times10^{-10})\right]$$
$$T \cong 7.02\times10^{-3}$$

(b) $$T \cong 16\left(\frac{0.12}{1.2}\right)\left(1-\frac{0.12}{1.2}\right) \times \exp\left[-2(5.3236\times10^9)(25\times10^{-10})\right]$$
$$T \cong 3.97\times10^{-12}$$

Ex 2.6

From Example 2.6, we have
$$E_n = \frac{-13.58}{(11.7)^2 n^2} = \frac{-0.0992}{n^2} \text{ eV}$$
$$E_1 = -99.2 \text{ meV},\ E_2 = -24.8 \text{ meV},$$
$$E_3 = -11.0 \text{ meV}$$

Test Your Understanding

TYU 2.1

(a) $$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{8\times10^{-10}}$$
$$= 1.318\times10^{-25} \text{ kg-m/s}$$

(b) $$\Delta E = \frac{dE}{dp}\cdot\Delta p = \left[\frac{d}{dp}\left(\frac{p^2}{2m}\right)\right]\cdot\Delta p$$
$$= \frac{2p}{2m}\cdot\Delta p = \frac{p\Delta p}{m}$$
$$\Delta E = \frac{(1.2\times10^{-23})(1.318\times10^{-25})}{9.11\times10^{-31}}$$
$$= 1.735\times10^{-18} \text{ J or } = 10.85 \text{ eV}$$

TYU 2.2

(a) $$\Delta E = (0.8)(1.6\times10^{-19}) = 1.28\times10^{-19} \text{ eV}$$
$$\Delta t = \frac{\hbar}{\Delta E} = \frac{1.054\times10^{-34}}{1.28\times10^{-19}} = 8.23\times10^{-16} \text{ s}$$

(b) Same as part (a), $\Delta t = 8.23\times10^{-16}$ s

TYU 2.3

(a) $$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$
$$= \sqrt{\frac{2(9.11\times10^{-31})(0.8-0.1)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}}$$
$$= 4.286\times10^9 \text{ m}^{-1}$$
$$T \cong 16\left(\frac{0.1}{0.8}\right)\left(1-\frac{0.1}{0.8}\right) \times \exp\left[-2(4.2859\times10^9)(12\times10^{-10})\right]$$
$$T \cong 5.97\times10^{-5}$$

(b) $$k_2 = \sqrt{\frac{2(9.11\times10^{-31})(1.5-0.1)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}}$$
$$= 6.061\times10^9 \text{ m}^{-1}$$
$$T \cong 16\left(\frac{0.1}{1.5}\right)\left(1-\frac{0.1}{1.5}\right) \times \exp\left[-2(6.061\times10^9)(12\times10^{-10})\right]$$
$$T \cong 4.79\times10^{-7}$$

TYU 2.4

$$T = 5\times10^{-6} = 16\left(\frac{0.08}{0.8}\right)\left(1-\frac{0.08}{0.8}\right)\exp(-2k_2a)$$
so that $$\exp(+2k_2a) = 2.88\times10^5$$
$$2k_2a = 12.571$$
$$k_2 = \sqrt{\frac{2(9.11\times10^{-31})(0.8-0.08)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}}$$
$$= 4.3467\times10^9 \text{ m}^{-1}$$
Then
$$a = \frac{12.571}{2(4.3467\times10^9)} = 1.446\times10^{-9} \text{ m}$$
or $a = 14.46 \overset{o}{A}$
