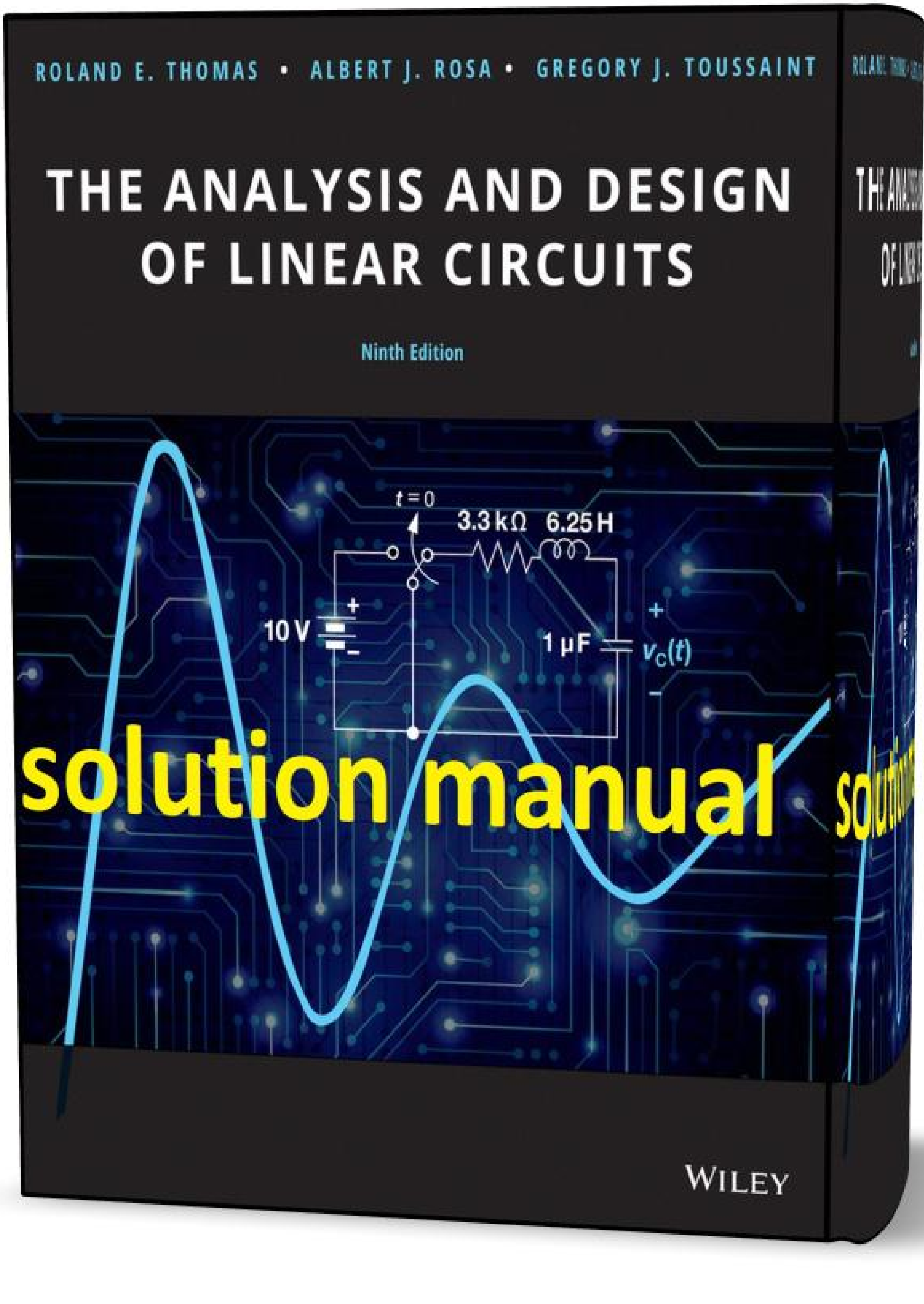

ROLAND E. THOMAS • ALBERT J. ROSA • GREGORY J. TOUSSAINT
THE ANALYSIS AND DESIGN OF LINEAR CIRCUITS
Ninth Edition
t = 0
3.3 kΩ
6.25 H
10 V
1 µF
$v_C(t)$
solution manual
WILEY

1 Introduction

Instructor's Supplement Problem Solutions

Problem 1–1. Electric power companies measure energy consumption in kilowatt-hours, denoted kWh. One kilowatt-hour is the amount of energy transferred by 1 kW of power in a period of 1 hour. A power company billing statement reports a user's total energy usage to be 1500 kWh. Find the number of joules used during the billing period.

We have the following relationships: 1 kWh = 1000 watt-hours, 1 watt = 1 joule/second, and 1 hour = 3600 seconds. So 1500 kWh = 1500 kilowatt-hours = 1500000 watt-hours = 1500000 (joules/second)(hours)(3600 second/hour) = 5.4×10^9 J = 5.4 GJ.

Problem 1–2. Which of the two entries is larger?

(a) 1000 microfarads or 0.0003333 F.

1000 microfarads = 1000×10^{-6} F = 1×10^{-3} F = 0.001 F > 0.0003333 F.

(b) 0.005×10^6 Hz or 66 kHz.

0.005×10^6 Hz = 5×10^3 Hz = 5 kHz < 66 kHz.

(c) 0.333 pC or 810 fC.

0.333 pC = 0.333×10^{-12} C = 333×10^{-15} C = 333 fC < 810 fC.

(d) 220 millihenries or 0.150 H.

220 millihenries = 220×10^{-3} H = 0.220 H > 0.150 H.

Problem 1–3. A wire carries a constant current of 30 μA. How many coulombs flow past a given point in the wire in 500 ms?

We know that 1 ampere is equivalent to 1 coulomb/second. Since the current is constant, if we multiply the current by the time, we get the charge flowing past a point over that period of time. We can calculate $q = i \times t = 30\ \mu\text{A} \times 500$ ms = (30×10^{-6} coulombs/second)(500×10^{-3} seconds) = 15000×10^{-9} C = 15×10^{-6} C = 15 μC.

Problem 1–4. A cell-phone charger outputs 9.6 V and is protected by a 50 mA fuse. A 2-W cell phone is connected to it to be charged. Will the fuse blow?

If the current to the cell phone exceeds 50 mA, then the fuse will blow. The current is the power divided by voltage, $i = p/v$ = (2 W)/(9.6 V) = 208.33 mA. The current is greater than 50 mA, so the fuse will blow.

The following MATLAB code calculates the same answer.

```
p = 2;
v = 9.6;
fuse = 50e-3;
i = p/v
FuseBlows = i>fuse
```

Problem 1–5. A string of holiday lights is protected by a $\frac{1}{2}$-A fuse and has 100 LED lights, each of which is rated at 30 mW. How many strings can be connected end-to-end across a 120 V circuit without blowing a fuse?

Each string of lights increases the power by 100×30 mW = 3 W. The current is the power divided by the voltage, $i = p/v$. Correspondingly, the current increases by (3 W)/(120 V) = 25 mA for each string. Since each fuse can handle up to 0.5 A, divide the fuse rating by the current required for each string and round down to get the maximum number of strings. We have (0.5 A)/(0.025 A) = 20. The maximum number of strings is 20.

Problem 1–6. A new 6-V Alkaline lantern battery delivers 237.5 kJ of energy during its lifetime. How long will the battery last in an application that draws 20 mA continuously. Assume the battery voltage is constant.

The power delivered is the product of the voltage and current, $p = vi$ = (6 V)(20 mA) = 120 mW. A watt is a joule per second, so the application draws 120 mJ/s. Divide the capacity of the battery by the rate to get the total time, (237.5 kJ)/(120 mJ/s) = 1.9792 Ms = 549.77 h = 22.91 days.

Problem 1–7. In Figure P1–7 the voltage v_2 is 20 V and v_4 is 10 V. Find the voltage associated with each element.

Element 1 has a ground on each side of the device, so there is no change in voltage across the device and $v_1 = 0$ V. Element 2 has a drop of 20 V from left to right and the left side is grounded, so the node to the right of Element 2 has a voltage of $0 - 20 = -20$ V. That same node is above Element 3. Element 3 connects the node with a -20 V to ground, so its voltage is $v_3 = -20 - 0 = -20$ V. Element 4 has a voltage drop of 10 V from left to right and the left side has a voltage of -20 V, so the node on the right side has a voltage of $-20 - 10 = -30$ V. Element 5 connects a ground on the left to the node with a voltage of -30 V on the right, so its voltage is $v_5 = 0 - (-30) = 30$ V. In summary, we have $v_1 = 0$ V, $v_2 = 20$ V, $v_3 = -20$ V, $v_4 = 10$ V, and $v_5 = 30$ V.

Problem 1–8. Using the passive sign convention, the voltage across a device is $v(t) = 240\cos(314t)$ V and the current through the device is $i(t) = 4\sin(314t)$ A. Using MATLAB, create a short script (m-file) to assign a value to the time variable, t, and then calculate the voltage, current, and power at that time. Run the script for $t = 5$ ms and $t = 10$ ms and for each result state whether the device is absorbing or delivering power.

The following MATLAB code provides the solution.

```
t = 5e-3
vt = 240*cos(314*t)
it = 4*sin(314*t)
pt = vt*it
Absorbing = pt>0

t = 10e-3
vt = 240*cos(314*t)
it = 4*sin(314*t)
pt = vt*it
Absorbing = pt>0
```

The corresponding MATLAB output is shown below.

```
t = 5.0000e-003
vt = 191.1184e-003
it = 4.0000e+000
pt = 764.4734e-003
Absorbing = 1

t = 10.0000e-003
vt = -239.9997e+000
it = 6.3706e-003
pt = -1.5289e+000
Absorbing = 0
```

The following table summarizes the results.

t (ms)	$v(t)$ (V)	$i(t)$ (A)	$p(t)$ (W)	Absorbing/Delivering
5	0.191	4.00	0.764	Absorbing
10	-240.00	0.00637	-1.529	Delivering

Problem 1–9. Computer Data Sheet (A). A manufacturer's data sheet for a notebook computer lists the power supply requirements as 7.5 A @ 5 V, 2 A @ 15 V, 2.5 A @ -15 V, 2.25 A @ -5 V and 0.5 A @ 12 V. The data sheet also states that the overall power consumption is 115 W. Are these data consistent? Explain.

Compute the power associated with each of the five requirements by multiplying the voltage and current together. The resulting values are 37.5 W, 30 W, -37.5 W, -11.25 W, and 6 W. In this case, it is not reasonable for the computer to supply power back to the power supply, so the negative powers are *not* a correct interpretation of the passive sign convention. We should take the absolute value of the individual powers to determine the total power requirement for the computer. Summing the magnitudes of the powers yields a total of 122.25 W, which is greater than the stated power consumption of 115 W. The stated power consumption is probably a reasonable value, based on the performance of the computer and the fact that, typically, the power supply with not have to deliver the maximum values constantly.

2 Basic Circuit Analysis

Instructor's Supplement Problem Solutions

Problem 2–1. A model railroader wants to be able to electrically throw a rail switch R_{Switch} from two different locations. He designs the circuit in Figure P2–1 using two single-pole double throw switches. Will it work? Explain.

The circuit will work as intended to power R_{Switch} from two different locations. If the railroader is at Location A, regardless of the setting of the switch at Location B, the railroader can operate Switch A to either make a complete circuit or an open circuit. The same options are available to the railroader at Location B, regardless of the setting of the switch at Location A.

Problem 2–2. The conductance of a particular semiconductor resistor is 0.05 mS. Find the current through the resistor when connected across a 1.5-V source.

Using the conductance version of Ohm's law, we have $i = Gv = (0.05 \times 10^{-3}\text{ S})(1.5\text{ V}) = 75\ \mu\text{A}$.

Problem 2–3. The i-v characteristic of a nonlinear resistor is $v = 82i + 0.17i^3$.

(a) Calculate v and p for $i = \pm 0.5$, ± 1, ± 2, ± 5, and ± 10 A.

Since there are 10 values for the current to examine, this problem is best solved with MATLAB. We will solve the problem for one current and then automate the process. Given $i = +0.5$ A, use the expression to calculate the voltage as $v = 82i + 0.17i^3 = (82)(0.5) + (0.17)(0.5)^3 = 41.0213$ V. We can now calculate the power as $p = vi = (41.0213)(0.5) = 20.5106$ W. To compute the other solutions, we use MATLAB as shown below.

```
ii = [-10, -5, -2, -1, -0.5, 0.5, 1, 2, 5, 10];
v = 82*ii + 0.17*ii.^3;
p = v.*ii;
Results = [ii' v' p']
```

The corresponding MATLAB output is shown below.

```
Results =
   -10.0000e+000  -990.0000e+000      9.9000e+003
    -5.0000e+000  -431.2500e+000      2.1563e+003
    -2.0000e+000  -165.3600e+000    330.7200e+000
    -1.0000e+000   -82.1700e+000     82.1700e+000
  -500.0000e-003   -41.0213e+000     20.5106e+000
   500.0000e-003    41.0213e+000     20.5106e+000
     1.0000e+000    82.1700e+000     82.1700e+000
     2.0000e+000   165.3600e+000    330.7200e+000
     5.0000e+000   431.2500e+000      2.1563e+003
    10.0000e+000   990.0000e+000      9.9000e+003
```

The results are summarized in the following table.

i (A)	v (V)	p (W)
-10	-990.00	9900.00
-5	-431.25	2156.30
-2	-165.36	330.72
-1	-82.17	82.17
-0.5	-41.02	20.51
0.5	41.02	20.51
1	82.17	82.17
2	165.36	330.72
5	431.25	2156.30
10	990.00	9900.00

(b) Find the maximum error in v when the device is treated as a 82-Ω linear resistance on the range $|i| < 0.5$ A.

The term $0.17i^3$ makes the expression for v nonlinear and represents the difference between the actual device and an 82-Ω resistor. The maximum error will occur when the absolute value of the cubic term is maximized. That occurs at the extreme values for i, which are ±0.5 A, in this case. At +0.5 A, the actual voltage is 41.0213 V and the voltage across a 82-Ω resistor is exactly 41 V. The error is 21.3 mV. At −0.5 A, the actual voltage is −41.0213 V and the voltage across a 82-Ω resistor is exactly −41 V, so the error is −21.3 mV. In both cases, the percentage of error is 0.052%, which is exceptionally small.

Problem 2–4. A certain type of film resistor is available with resistance values between 10 Ω and 100 MΩ. The maximum ratings for all resistors of this type are 500 V and 0.25 W. Show that the voltage rating is the controlling limit for $R > 1\,\text{M}\Omega$, and that the power rating is the controlling limit when $R < 1\,\text{M}\Omega$.

The power dissipated by a resistor can be expressed as $p = v^2/R$. Solving for the resistance, we have $R = v^2/p$. With both maximum ratings applied, the resistance is $R = (500)^2/(0.25) = 1\,\text{M}\Omega$. Therefore at $R = 1\,\text{M}\Omega$ there are no issues with either type of rating. If the resistance increases above 1 MΩ, then using $R = v^2/p$, the maximum power must be less than 0.25 W. Therefore, the resistor will never dissipate 0.25 W and the voltage rating will be the only active constraint. If the resistance is less than 1 MΩ, then the maximum voltage must be less than 500 V and power rating will be the only active constraint.

Problem 2–5. For the circuit in Figure P2–5:

(a) Identify the nodes and at least two loops.

The circuit has three nodes and three loops. The nodes are labeled A, B, and C. The first loop contains elements 1 and 2, the second loop contains elements 2, 3, and 4, and the third loop contains elements 1, 3 and 4.

(b) Identify any elements connected in series or in parallel.

Elements 3 and 4 are connected in series. Elements 1 and 2 are connected in parallel.

(c) Write KCL and KVL connection equations for the circuit.

The KCL equations are

$$\begin{aligned}
&\text{Node A} \quad -i_1 - i_2 - i_3 = 0 \\
&\text{Node B} \quad i_3 - i_4 = 0 \\
&\text{Node C} \quad i_1 + i_2 + i_4 = 0
\end{aligned}$$

The KVL equations are

$$\begin{aligned}
&\text{Loop 1 2} \quad -v_1 + v_2 = 0 \\
&\text{Loop 2 3 4} \quad -v_2 + v_3 + v_4 = 0 \\
&\text{Loop 1 3 4} \quad -v_1 + v_3 + v_4 = 0
\end{aligned}$$

Problem 2–6. In many circuits the ground is often the metal case that houses the circuit. Occasionally a failure occurs whereby a wire connected to a particular node touches the case causing that node to become connected to ground. Suppose that in Figure P2–6 node C accidentally touches ground. If $v_2 = 20$ V, $v_3 = -20$ V, and $v_4 = 6$ V, find v_1, v_5, and v_6.

If Node C is connected to ground, then element 5 is connected to ground on both sides, so its voltage is $v_5 = 0$ V. The KVL equations are

$$\begin{aligned}
&\text{Loop 1 3 2} \quad -v_1 - v_3 + v_2 = 0 \\
&\text{Loop 2 4 5} \quad -v_2 + v_4 + v_5 = 0 \\
&\text{Loop 3 6 4} \quad v_3 + v_6 - v_4 = 0
\end{aligned}$$

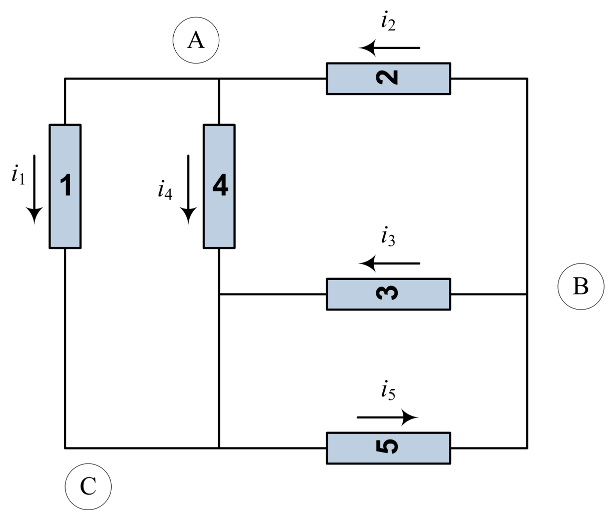
Using the first equation, we can solve for $v_1 = v_2 - v_3 = 20 + 20 = 40$ V. Using the third equation, we can solve $v_6 = v_4 - v_3 = 6 + 20 = 26$ V. However, if we apply the second equation and $v_5 = 0$ V, we get $v_2 = v_4$, which is not consistent with the given voltages. If node C is connected to ground, the given voltages are not possible.

Problem 2–7. The KCL equations for a three-node circuit are:

$$\begin{aligned} &\text{Node A} \quad -i_1 + i_2 - i_4 = 0 \\ &\text{Node B} \quad -i_2 - i_3 + i_5 = 0 \\ &\text{Node C} \quad i_1 + i_3 + i_4 - i_5 = 0 \end{aligned}$$

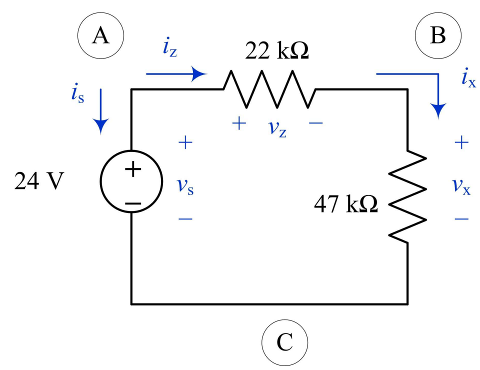
Draw the circuit diagram and indicate the reference directions for the element currents.

There are many equivalent diagrams to solve this problem. One possible solution is shown in the figure below.



Problem 2–8. For the circuit in Figure P2–8, write a complete set of connection and element constraints and then find v_x and i_x.

Start by annotating the circuit as shown below.



There is one KVL connection equation:

$$-v_s + v_z + v_x = 0$$

We can write KCL connection equations for Nodes A, B, and C:

$$\begin{aligned} -i_s - i_z &= 0 \\ i_z - i_x &= 0 \\ i_s + i_x &= 0 \end{aligned}$$

The element constraints are:

$$\begin{aligned} v_s &= 24\,\text{V} \\ v_z &= (22\,\text{k}\Omega)i_z \\ v_x &= (47\,\text{k}\Omega)i_x \end{aligned}$$

Solving the second KCL equation, we find $i_x = i_z$. We can now solve for v_x and i_x as follows:

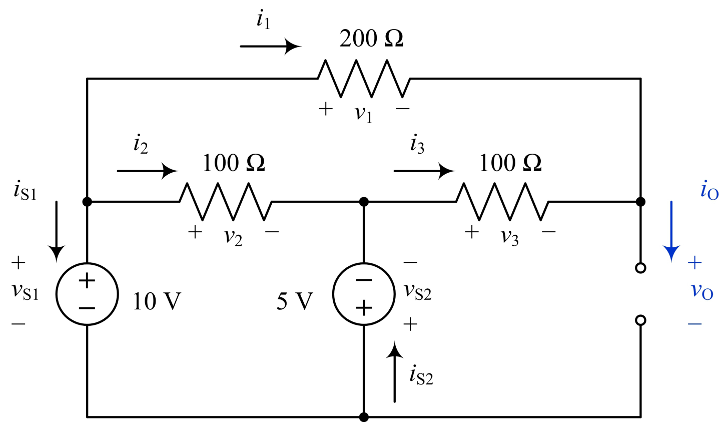
$$\begin{aligned} v_s &= v_x + v_z \\ 24 &= (47\,\text{k}\Omega)i_x + (22\,\text{k}\Omega)i_z \\ 24 &= (47\,\text{k}\Omega)i_x + (22\,\text{k}\Omega)i_x \quad = \quad (69\,\text{k}\Omega)i_x \\ i_x &= 347.826\,\mu\text{A} \\ v_x &= (47\,\text{k}\Omega)i_x \quad = \quad (47\,\text{k}\Omega)(347.826\,\mu\text{A}) = 16.3478\,\text{V} \end{aligned}$$

Problem 2–9. Find v_x and i_x in Figure P2–9.

In the circuit, 0.5 A flows through the 10-Ω resistor in the center. The voltage drop across this resistor is $v = Ri = (10\,\Omega)(0.5\,\text{A}) = 5\,\text{V}$. The 10-Ω resistor is connected in parallel to the 5-Ω resistor, so they have the same voltage drop. The associated KVL equation verifies this fact. With 5 V across the 5-Ω resistor, the current is $i_x = v/R = (5\,\text{V})/(5\,\Omega) = 1$ A. KCL at the top node requires that the current entering the node equals the current leaving the node. Since we have $0.5 + 1.0 = 1.5$ A leaving the node, 1.5 A must enter the node through the 5 Ω resistor. The voltage drop across the 5 Ω resistor is $v = Ri = (5\,\Omega)(1.5\,\text{A}) = 7.5$ V. We can now write a KVL equation around the first loop to get $-v_x + 7.5 + 5 = 0$, which implies $v_x = 12.5$ V.

Problem 2–10. Find v_O in the circuit of Figure P2–10.

Assign a voltage and current variable to every element, as shown in the figure below.



The KVL equations for the circuit are

$$\begin{aligned} -v_{S1} + v_1 - v_3 - v_{S2} &= 0 \\ -v_{S1} + v_2 - v_{S2} &= 0 \\ v_{S2} + v_3 + v_O &= 0 \end{aligned}$$

The KCL equations for the circuit are

$$\begin{aligned} -i_1 - i_2 - i_{S1} &= 0 \\ i_2 - i_3 + i_{S2} &= 0 \\ i_1 + i_3 &= 0 \end{aligned}$$

The component equations for the circuit are

$$\begin{aligned} v_1 &= R_1 i_1 \\ v_2 &= R_2 i_2 \\ v_3 &= R_3 i_3 \end{aligned}$$

Solve the second KVL equation to determine v_2:

$$v_2 = v_{S1} + v_{S2} = 10 + 5 = 15\,\text{V}$$

Use the first KVL equation, the component equations, and the third KCL equation to solve for i_1:

$$\begin{aligned} v_1 - v_3 &= v_{S1} + v_{S2} \\ R_1 i_1 - R_3 i_3 &= v_{S1} + v_{S2} \\ i_3 &= -i_1 \\ R_1 i_1 + R_3 i_1 &= v_{S1} + v_{S2} \\ (R_1 + R_3) i_1 &= v_{S1} + v_{S2} \\ i_1 &= \frac{v_{S1} + v_{S2}}{R_1 + R_3} = \frac{10 + 5}{100 + 200} = 50\,\text{mA} \end{aligned}$$

Applying the third KCL equation again, we have $i_3 = -i_1 = -50\,\text{mA}$. Solve for $v_3 = R_3 i_3 = (100\,\Omega)(-50\,\text{mA}) = -5\,\text{V}$. Apply the third KVL equation to find v_O:

$$v_O \;=\; -v_{S2} - v_3 \;=\; -5 + 5 = 0\,\text{V}$$

Problem 2–11. In Figure P2–11, $i_x = 0.33$ mA. Find the value of R.

Label the left 10-kΩ resistor as R_x, with voltage v_x. Label the unknown resistor as R_1, with current i_1 flowing down. Label the right 10-kΩ resistor as R_2 with the current flowing right to left. Apply the passive sign convention to label the voltages. Use Ohm's law to solve for $v_x = R_x i_x = (10\,\text{k}\Omega)(0.33\,\text{mA}) = 3.3$ V. Write the KVL equation for the left side as $-4\text{ V} + v_x + v_1 = 0$ and solve for v_1 as $v_1 = 4\,\text{V} - v_x = 4\,\text{V} - 3.3\,\text{V} = 0.7$ V. Write the KVL equation for the right side as $-v_1 - v_2 + 15\text{ V} = 0$ and solve for v_2 as $v_2 = 15\,\text{V} - v_1 = 15\,\text{V} - 0.7\,\text{V} = 14.3$ V. Use Ohm's law to solve for $i_2 = v_2/R_2 = (14.3\,\text{V})(10\,\text{k}\Omega) = 1.43$ mA. Write the KCL equation for the center node as $-i_1 + i_2 + i_x = 0$ and solve for i_1 as $i_1 = i_2 + i_x = 1.43\,\text{mA} + 0.33\,\text{mA} = 1.76$ mA. Use Ohm's law to find $R = R_1 = v_1/i_1 = (0.7\,\text{V})/(1.76\,\text{mA}) = 397.7\,\Omega$.

Problem 2–12. Equivalent resistance is defined at a particular pair of terminals. In Figure P2–12, the same circuit is looked at from two different terminal pairs. Find the equivalent resistances R_{EQ1} and R_{EQ2} in Figure P2–12. Note that in calculating R_{EQ2} the 33-kΩ resistor is connected to an open circuit and therefore does not affect the calculation.

For R_{EQ1}, ignore the two terminals on the right and collapse the circuit from right to left. The 10-kΩ, 22-kΩ, and 15-kΩ resistors are in series; that result is in parallel with the 56-kΩ resistor; and that result is in series with the 33-kΩ resistor. We can calculate the equivalent resistance as follows:

$$R_{EQ1} = 33 + [56 \parallel (10 + 22 + 15)] = 33 + [56 \parallel 47] = 33 + 25.55 = 58.55\,\text{k}\Omega$$

For R_{EQ2}, ignore the two terminals on the left and the 33-kΩ resistor. Collapse the circuit from left to right. The 10-kΩ, 56-kΩ, and 15-kΩ resistors are in series and that result is in parallel with the 22-kΩ resistor. We can calculate the equivalent resistance as follows:

$$R_{EQ2} = [(10 + 56 + 15) \parallel 22] = [81 \parallel 22] = 17.30\,\text{k}\Omega$$

Problem 2–13. Show how the circuit in Figure P2–13 could be connected to achieve a resistance of 100 Ω, 200 Ω, 150 Ω, 50 Ω, 25 Ω, 33.3 Ω, and 133.3 Ω.

For 100 Ω, we need a single 100-Ω resistor, which is a connection between terminals A and D. For 200 Ω, we need two 100-Ω resistors in series, which is a connection between terminals A and B. For 150 Ω, we need a 100-Ω resistor in series with a 50-Ω resistor, which is a connection between terminals A and C. For 50 Ω, we need a single 50-Ω resistor, which is a connection between terminals C and D. We can get 25 Ω by connecting the two 100-Ω resistors in parallel, which yields 50 Ω, and then connecting that result in parallel with the 50-Ω resistor, to get 25 Ω. The required combination is to connect the A, B, and C terminals together on one side and have the D terminal on the other. For 33.3 Ω, connect the 100 and 50-Ω resistors in parallel, which requires B and C to be connected on one side and the D terminal on the other. Finally, to get 133.3 Ω, connect a 100-Ω resistor in series with a parallel combination of a 100 and a 50-Ω resistor. This requires a connection to the A terminal on one side and the B and C terminals connected on the other. The following table summarizes the results.

Resistance (Ω)	Terminal 1	Terminal 2
100	A	D
200	A	B
150	A	C
50	C	D
25	A+B+C	D
33.3	B+C	D
133.3	A	B+C

Problem 2–14. Select a value of R_L in Figure P2–14 so that $R_{EQ} = 15$ kΩ. Repeat for $R_{EQ} = 11$ kΩ.

Create an expression for R_{EQ} in terms of R_L and then solve for R_L. Use the new expression to find the appropriate values for R_L for the given values of R_{EQ}. All resistance are in kilohms.

$$\begin{aligned}
R_{EQ} &= 22 \parallel (22 + R_L) = \frac{22(22 + R_L)}{22 + 22 + R_L} = \frac{484 + 22R_L}{44 + R_L} \\
R_{EQ}(44 + R_L) &= 484 + 22R_L \\
44R_{EQ} + R_{EQ}R_L &= 484 + 22R_L \\
R_{EQ}R_L - 22R_L &= 484 - 44R_{EQ} \\
(R_{EQ} - 22)R_L &= 484 - 44R_{EQ} \\
R_L &= \frac{484 - 44R_{EQ}}{R_{EQ} - 22}
\end{aligned}$$

For $R_{EQ} = 15$ kΩ, we have

$$R_L = \frac{484 - (44)(15)}{15 - 22} = \frac{-176}{-7} = 25.14\,\text{k}\Omega$$

For $R_{EQ} = 11$ kΩ, we have

$$R_L = \frac{484 - (44)(11)}{11 - 22} = \frac{0}{-11} = 0\,\text{k}\Omega$$

Problem 2–15. In Figure P2–15, the i-v characteristic of network N is $v + 50i = 5$ V. Find the equivalent practical current source for the network.

When the circuit is open between nodes A and B, there is no current, $i = 0$ A, and the voltage must be $v = 5$ V in order to satisfy the i-v characteristic. When a short is placed between nodes A and B, the voltage is zero, $v = 0$ V, and the current is $i = 100$ mA in order to satisfy the i-v characteristic. The corresponding practical current source will have a current $i_S = 100$ mA and a parallel resistance $R = v_S/i_S = (5\,\text{V})/(100\,\text{mA}) = 50\ \Omega$.

Problem 2–16. What is the range of R_{EQ} in Figure P2–16?

The poteniometer can range from 0 Ω to 15 kΩ and it is in parallel with a 15-kΩ resistor. That parallel combination is in series with a 10-kΩ resistor. When the poteniometer has a value of 0 Ω, it shorts out the 15-kΩ resistor, so only the 10-kΩ resistor is active and $R_{EQ} = 10$ kΩ. When the poteniometer has a value of 15 kΩ, the parallel combination is $15 \parallel 15 = 7.5$ kΩ. That result is in series with the 10-kΩ resistor, so we have $R_{EQ} = 10 + 7.5 = 17.5$ kΩ. R_{EQ} varies between 10 and 17.5 kΩ.

Problem 2–17. Use voltage division in Figure P2–17 to obtain an expression for v_L in terms of R, R_L, and v_S.

The two right resistors are in parallel and the voltage v_L appears across that combination. Combine the parallel resistors and then use voltage division to develop the expression for v_L.

$$\begin{aligned}
R_{EQ} &= R \parallel R_L = \frac{RR_L}{R + R_L} \\
v_L &= \left[\frac{R_{EQ}}{R + R_{EQ}}\right] v_S = \left[\frac{\dfrac{RR_L}{R + R_L}}{R + \dfrac{RR_L}{R + R_L}}\right] v_S = \left[\frac{RR_L}{R^2 + RR_L + RR_L}\right] v_S \\
v_L &= \frac{R_L\, v_S}{R + 2R_L}
\end{aligned}$$

Problem 2–18. Use current division in Figure P2–18 to find an expression for v_L in terms of R, R_L, and i_S.

Combine the two right resistors in series to get an equivalent resistance $R_{EQ} = R + R_L$. Apply the two-path current division rule to solve for the current through R_L.

$$i_L = \frac{R}{R + R_{EQ}}(i_S) = \frac{R}{R + R + R_L}(i_S) = \frac{R}{2R + R_L}(i_S)$$

Apply Ohm's law to solve for the voltage v_L

$$v_L = R_L i_L = \frac{R R_L i_S}{2R + R_L}$$

Problem 2–19. Find the range of values of v_O in Figure P2–19.

If we combine the right resistor with the poteniometer in parallel, we can use voltage division to solve for v_O. The poteniometer takes on values from 0 Ω to 1.5 kΩ. When the poteniometer is 0 Ω, the output is shorted out and the voltage is $v_O = 0$ V. When the poteniometer is 1.5 kΩ, the parallel combination is 1500 ∥ 1500 = 750 Ω. The output voltage is therefore

$$v_O = \frac{750}{1000 + 750}(12) = 5.14286\,\text{V}$$

Problem 2–20. (A) Ideally, a voltmeter has infinite internal resistance and can be placed across any device to read the voltage without affecting the result. A particular digital multimeter (DMM), a common laboratory tool, is connected across the circuit shown in Figure P2–20. The expected voltage was 10.2 V. However, the DMM reads 7.61 V. The large, but finite, internal resistance of the DMM was "loading" the circuit and causing a wrong measurement to be made. Find the value of the internal resistance R_M of this DMM.

Apply voltage division to find the equivalent resistance of the parallel combination of the 10-MΩ resistor with the DMM.

$$\begin{aligned}
7.61\,\text{V} &= \frac{R_{EQ}}{4.7 + R_{EQ}}(15\,\text{V}) \\
7.61(4.7 + R_{EQ}) &= 15 R_{EQ} \\
7.39 R_{EQ} &= 35.767 \\
R_{EQ} &= 4.83992\,\text{M}\Omega
\end{aligned}$$

Now use the expression for a parallel combination of resistors to find the internal resistance of the DMM.

$$\begin{aligned}
R_{EQ} &= \frac{10 R_M}{10 + R_M} \\
4.83992 &= \frac{10 R_M}{10 + R_M} \\
4.83992(10 + R_M) &= 10 R_M \\
5.16008 R_M &= 48.3992 \\
R_M &= 9.3795\,\text{M}\Omega
\end{aligned}$$

Problem 2–21. (D) Select a value of R_x in Figure P2–21 so that $v_L = 2$ V. Repeat for 4 V and 6 V. *Caution*: R_x must be positive.

The three resistors are connected in parallel, so they share the same voltage drop. Use the known resistor values and the voltage drop to find the currents through the 100-Ω and 50-Ω resistors and then apply KCL to find the current through R_x. Then compute the value of R_x using Ohm's law.

$$\begin{aligned} i_{100} &= \frac{2\,\text{V}}{100\,\Omega} = 20\,\text{mA} \\ i_{50} &= \frac{2\,\text{V}}{50\,\Omega} = 40\,\text{mA} \\ i_x &= 120\,\text{mA} - i_{100} - i_{50} = (120-20-40)\,\text{mA} = 60\,\text{mA} \\ R_x &= \frac{v_x}{i_x} = \frac{2\,\text{V}}{60\,\text{mA}} = 33.33\,\Omega \end{aligned}$$

We can use a similar approach for the other two output voltages. For $v_L = 4$ V, we have:

$$\begin{aligned} i_{100} &= \frac{4\,\text{V}}{100\,\Omega} = 40\,\text{mA} \\ i_{50} &= \frac{4\,\text{V}}{50\,\Omega} = 80\,\text{mA} \\ i_x &= 120\,\text{mA} - i_{100} - i_{50} = (120-40-80)\,\text{mA} = 0\,\text{mA} \end{aligned}$$

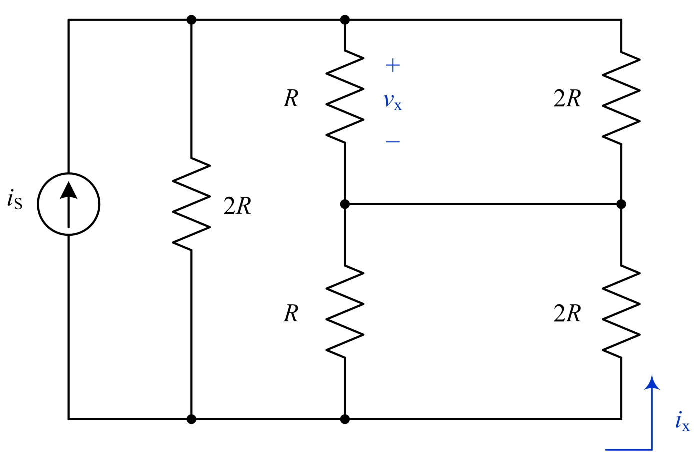
Since there is no current flowing through R_x, it must be an open circuit for $v_L = 4$ V. Now solve for $v_L = 6$ V.

$$\begin{aligned} i_{100} &= \frac{6\,\text{V}}{100\,\Omega} = 60\,\text{mA} \\ i_{50} &= \frac{6\,\text{V}}{50\,\Omega} = 120\,\text{mA} \\ i_x &= 120\,\text{mA} - i_{100} - i_{50} = (120-60-120)\,\text{mA} = -60\,\text{mA} \end{aligned}$$

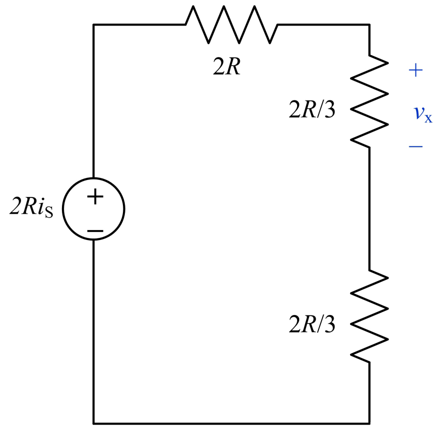
It is not possible for the resistor R_x to have a negative current and a positive voltage, since that would imply that it is supplying power rather than dissipating it. There is no valid solution for $v_L = 6$ V.

Problem 2–22. Use circuit reduction to find v_x and i_x in Figure P2–22.

The figure below displays a circuit that is electrically equivalent to the original circuit.



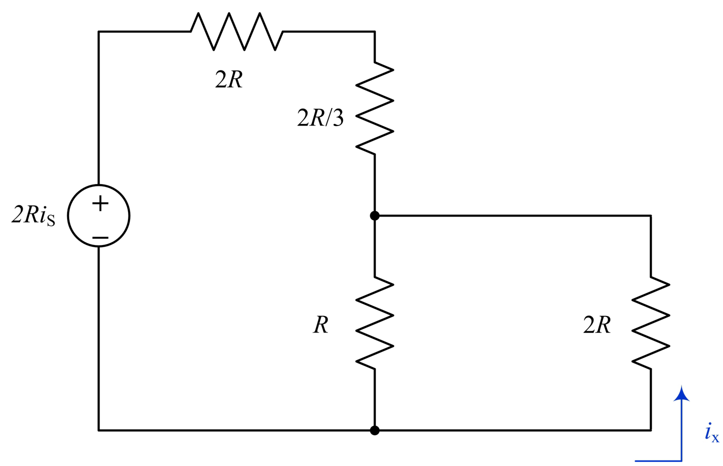
Combine the two sets of R and $2R$ resistors in parallel and perform a source transformation to get the circuit shown below, which retains v_x.



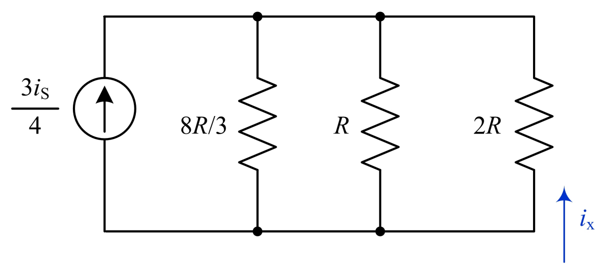
Apply voltage division to solve for v_x as follows

$$v_x = \frac{\frac{2R}{3}}{2R + \frac{2R}{3} + \frac{2R}{3}}(2Ri_S) = \frac{2}{6+2+2}(2Ri_S) = \frac{2Ri_S}{5}$$

To solve for i_x, start with the first circuit shown above, perform a source transformation, and combine only the top resistors in parallel to get an equivalent resistance of $R_{EQ1} = 2R/3$ as shown in the figure below.



Combine the two resistors in series to get an equivalent resistance of $R_{EQ2} = 8R/3$ in series with the voltage source. Perform another source transformation to get the circuit shown below.

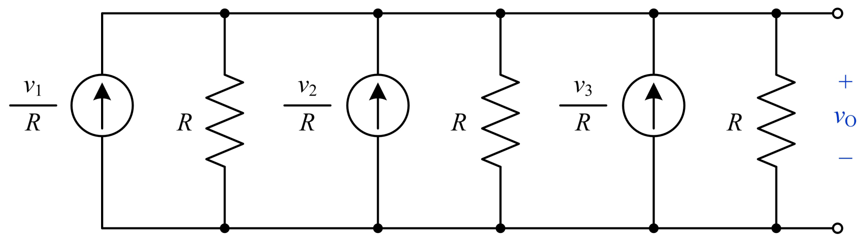


Apply current division to solve for i_x, noting that its direction is opposite that of the source, so it will be negative.

$$i_x = \frac{\frac{1}{2R}}{\frac{3}{8R} + \frac{1}{R} + \frac{1}{2R}}\left(\frac{-3i_S}{4}\right) = \frac{4}{3+8+4}\left(\frac{-3i_S}{4}\right) = \frac{-i_S}{5}$$

Problem 2–23. Use source transformations in Figure P2–23 to relate v_O to v_1, v_2, and v_3.

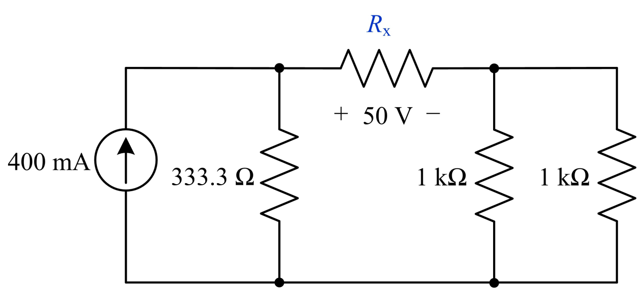
Perform a source transformation on each voltage source to get the equivalent circuit shown below.



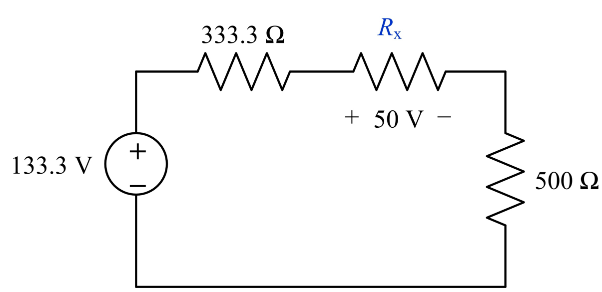
The current sources are in parallel, so they combine by summing to give an equivalent current source of $i_S = (v_1 + v_2 + v_3)/R$. The three resistors are in parallel, so they combine to yield an equivalent resistance of $R_{EQ} = R/3$. Apply Ohm's law to compute the output voltage $v_O = i_S R_{EQ} = (v_1 + v_2 + v_3)/3$.

Problem 2–24. Select R_x so that 50 V are across it in Figure P2–24.

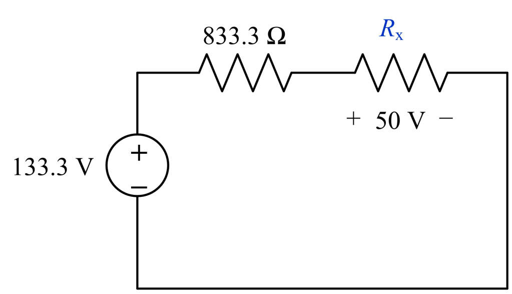
On the left side of the circuit, combine the 500-Ω and 1-kΩ resistors in parallel to get an equivalent resistance of 333.3 Ω. On the right side of the circuit, combine the two 500-Ω resistors in series. The resulting circuit is shown below.



Perform a source transformation on the left and combine the two 1-kΩ resistors in parallel on the right to get the equivalent circuit shown below.



Combine the 333.3-Ω and 500-Ω resistors in series to get the equivalent circuit shown below.



Apply voltage division to solve for R_x

$$\begin{aligned}
50\,\text{V} &= \frac{R_x}{833.3 + R_x}(133.3\,\text{V}) \\
50(R_x + 833.3) &= 133.3 R_x \\
83.3 R_x &= 41665 \\
R_x &= 500\,\Omega
\end{aligned}$$

Problem 2–25. A circuit is found to have the following element and connection equations:

$$\begin{aligned}
v_1 &= 24\,\text{V} & -v_1 + v_2 + v_3 &= 0 \\
v_2 &= 8000\,i_2 & -v_3 + v_4 + v_5 &= 0 \\
v_3 &= 5000\,i_3 & i_1 + i_2 &= 0 \\
v_4 &= 4000\,i_4 & -i_2 + i_3 + i_4 &= 0 \\
v_5 &= 16000\,i_5 & -i_4 + i_5 &= 0
\end{aligned}$$

Use MATLAB to solve for all of the unknown voltages and currents associated with this circuit. Sketch one possible schematic that matches the given equations.

There are many valid approaches to solve this problem using MATLAB. One way is to write the equations in matrix form and solve by inverting the matrix. Choose a vector of variables as

$$\mathbf{x} = \left[v_1\ v_2\ v_3\ v_4\ v_5\ i_1\ i_2\ i_3\ i_4\ i_5\right]^{\text{T}}$$

and write the equations in matrix form as follows.

$$\mathbf{A}\,\mathbf{x} = \mathbf{B}$$

where

$$\mathbf{A} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -8000 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -5000 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -4000 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -16000 \\
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 24 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve for the unknown values by calculating

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{B}$$

The following MATLAB code provides the solution.

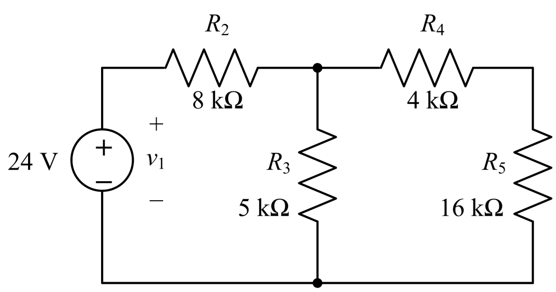
```
A = [1 0 0 0 0 0 0 0 0 0;
     0 1 0 0 0 0 -8000 0 0 0;
     0 0 1 0 0 0 0 -5000 0 0;
     0 0 0 1 0 0 0 0 -4000 0;
     0 0 0 0 1 0 0 0 0 -16000;
```

```
      -1  1  1  0  0  0  0  0  0  0;
      0  0  -1  1  1  0  0  0  0  0;
      0  0  0  0  0  1  1  0  0  0;
      0  0  0  0  0  0  -1  1  1  0;
      0  0  0  0  0  0  0  0  -1  1];
B = [24  0  0  0  0  0  0  0  0  0]';
x = A\B
```

The corresponding MATLAB output is shown below.

```
x =
   24.0000
   16.0000
    8.0000
    1.6000
    6.4000
   -0.0020
    0.0020
    0.0016
    0.0004
    0.0004
```

The figure below displays one possible circuit that corresponds to the given equations.

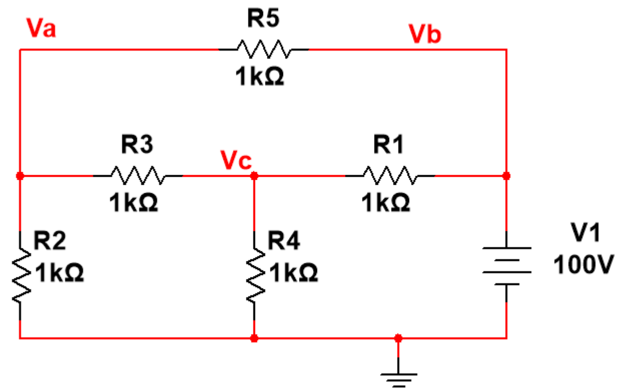


In summary, the voltages and currents are as follows

$v_1 = 24\,\text{V}$	$i_1 = -2\,\text{mA}$
$v_2 = 16\,\text{V}$	$i_2 = 2\,\text{mA}$
$v_3 = 8\,\text{V}$	$i_3 = 1.6\,\text{mA}$
$v_4 = 1.6\,\text{V}$	$i_4 = 0.4\,\text{mA}$
$v_5 = 6.4\,\text{V}$	$i_5 = 0.4\,\text{mA}$

Problem 2–26. The circuit of Figure P2–26 is called a "*bridge-T*" circuit. Use Multisim to find all of the voltages and currents in the circuit.

The Multisim circuit and the corresponding results shown below provide the solution.



Problem02_80
DC Operating Point Analysis

	Variable	Operating point value
1	V(va)	50.00000
2	V(va)-V(vb)	-50.00000
3	V(va)-V(vc)	0.00000e+000
4	V(vb)	100.00000
5	V(vb)-V(vc)	50.00000
6	V(vc)	50.00000
7	I(R1)	-50.00000 m
8	I(R2)	50.00000 m
9	I(R3)	0.00000e+000
10	I(R4)	50.00000 m
11	I(R5)	-50.00000 m
12	I(V1)	-100.00000 m

Note that resistor R_3 has no current following through it.

Problem 2–27. Transistor Biasing (**D**)

The circuit shown in Figure P2–27 is a typical biasing arrangement for a BJT-type transistor. The actual transistor for this problem can be modeled as 0.7-V battery in series with a 200-kΩ resistor. Biasing allows signals that have both positive and negative variations to be properly amplified by the transistor. Select the two biasing resistors R_A and R_B so that 3 V ±0.1 V appears across R_B.

Label the voltage across the 200-kΩ resistor as v_T. Write a KVL equation with resistor R_B and the transistor to get

$$-3 + 0.7 + v_T = 0$$

Solve for $v_T = 2.3$ V. The current through the 200-kΩ resistor is $i_T = (2.3\,\text{V})/(200\,\text{k}\Omega) = 11.5\ \mu\text{A}$. The voltage across R_B is 3 V, which makes the voltage across $R_A = 15 - 3 = 12$ V. As part of the design, choose the current through R_B to be approximately equal to the transistor current of 11.5 μA. The required resistance is $R_B = (3\,\text{V})/(11.5\,\mu\text{A}) = 260.87$ kΩ. A standard resistor value that is close is 270 kΩ. With $R_B = 270$ kΩ, the current through R_B is $i_B = (3\,\text{V})/(270\,\text{k}\Omega) = 11.11\ \mu\text{A}$. Applying KCL, the current through R_A is the sum of the currents through R_B and the transistor, which yields $i_A = 11.5 + 11.11 = 22.61\ \mu\text{A}$. The required resistance is $R_A = (12\,\text{V})/(22.61\,\mu\text{A}) = 530.71$ kΩ. Create R_A by combining a 330-kΩ and two 100-kΩ resistors in series.