# Fundamentals of Dynamics and Control of Space Systems

# Solution Manual

**Krishna Dev Kumar**


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Cover page: Full view of the International Space Station as photographed from the Space Shuttle Discovery during the STS-114 Return to Flight mission, following the undocking of the two spacecraft. (Courtesy of NASA)

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# Preface

This Solution Manual is prepared to accompany and supplement the author's text "Fundamentals of Dynamics and Control of Space Systems" by Krishna Dev Kumar, 2012. It contains detailed solutions for most problems in the textbook.

**September 3, 2012** **Krishna Kumar**

# Contents

# Chapter 2

# Kinematics, Momentum and Energy

## Problem Set 2

**2.1** The coordinate frames used in studying the dynamics of a spacecraft are as follows:

a) Inertial reference frame,

b) Orbital reference frame,

c) Perifocal reference frame,

c) Satellite body-fixed reference frame.

**2.2** The inertial frames are those coordinate frames that are nonrotating and nonaccelerating frames. The inertial frames are relevant because in applying the Newton's second law of motion

$$\vec{F} = m\frac{d\vec{V}}{dt} \tag{2.1}$$

to derive the equation of motion of a system, the velocity $\vec{V}$ and the corresponding acceleration $d\vec{V}/dt$ in the right-hand side of the above equation are to measured with respect to an inertial frame of reference.

An Earth-fixed frame is not an inertial frame as it is spinning about its axis with a period of 24 hour. When viewed from space, the point on the surface of the earth moves in a circle as the earth spins on its axis. Thus, it is accelerating with an centripetal acceleration of $r\omega^2$, where $r$ is the position of the point of the Earth center of mass and $\omega$ is the rate of spin of the Earth. With the earth a point on its surface also orbits the Sun. With the solar system, it orbits the center of the galaxy. Thus, the Earth-fixed frame is an accelerating frame and not an inertial frame.

We consider just the effect of the spinning motion of the Earth and therefore the inertial acceleration can be written as

$$\left.\frac{d\vec{V}}{dt}\right|_{inertial} = \left.\frac{d\vec{V}}{dt}\right|_{body} + \vec{\omega} \times \vec{V}_{body} \tag{2.2}$$

The corresponding error in considering an Earth-fixed frame as an inertial frame is

$$\text{Error} = \left.\frac{d\vec{V}}{dt}\right|_{inertial} - \left.\frac{d\vec{V}}{dt}\right|_{body} = \vec{\omega} \times \vec{V}_{body} \tag{2.3}$$

The Earth's spin rate $\omega$ is

$$\begin{aligned}\vec{\omega} &= \omega_k \hat{k} = -\frac{2\pi}{T}\hat{k} \\ &= -\frac{2\pi}{24 \times 3600}\hat{k} = 7.275 \times 10^{-5}\hat{k}\end{aligned} \tag{2.4}$$

where $\hat{k}$ is a unit vector along the z-direction as taken for the aircraft body-fixed frame.

The order of magnitude error would be $10^{-4} \times V_{body}$. As this magnitude is usually very small when compared to the magnitude of other relevant accelerations like the gravitational acceleration, which is 9.81 m/s$^2$, and we often treat the Earth-fixed frame as an inertial frame. when solving problems.

**2.3** The inertial position vectors for spacecraft $m_1$ and $m_2$ are

$$\vec{R}_1 = \vec{R} - \gamma\vec{L} \tag{2.5}$$

$$\vec{R}_2 = \vec{R} + (1-\gamma)\vec{L} \tag{2.6}$$

where $\gamma = m_2/(m_1 + m_2)$. The corresponding magnitudes are

$$R_1 = [R^2 + \gamma^2 L^2 - 2\gamma\vec{R}\cdot\vec{L}]^{1/2} \tag{2.7}$$

$$R_2 = [R^2 + (1-\gamma)^2 L^2 + 2(1-\gamma)\vec{R}\cdot\vec{L}]^{1/2} \tag{2.8}$$

where $L = L_0 + vt$. The nomenclature $L_o$ defines the initial length of the cable while $v$ is the speed by which the length of the cable varies.

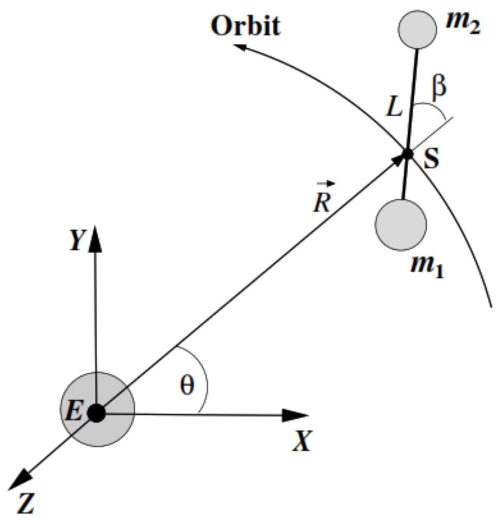


Figure 2.1: A dumbbell satellite system undergoing in-plane libration.

Expressing $\vec{R}$ and $\vec{L}$ in terms of unit vectors of the respective coordinate frames as

$$\vec{R} = R\hat{i}_o, \quad \vec{L} = L\hat{i} \tag{2.9}$$

Applying the transformation between coordinate frames $S - x_o y_o z_o$ and $S - xyz$, we get

$$\hat{i}_o \cdot \hat{i} = cos\beta \tag{2.10}$$

and using this relation in Eqs. (2.7) and (2.8), we have the inertial positions of the spacecraft $m_1$ and $m_2$ as

$$R_1 = [R^2 + \gamma^2 L^2 - 2\gamma RLcos\beta]^{1/2} \tag{2.11}$$

$$R_2 = [R^2 + (1-\gamma)^2 L^2 + 2(1-\gamma)RLcos\beta]^{1/2} \tag{2.12}$$

The inertial velocity vectors for spacecraft $m_1$ and $m_2$ are

$$\vec{V}_1 = \dot{\vec{R}}_1 = \dot{\vec{R}} - \gamma\dot{\vec{L}} \tag{2.13}$$

$$\vec{V}_2 = \dot{\vec{R}}_2 = \dot{\vec{R}} + (1-\gamma)\dot{\vec{L}} \tag{2.14}$$

The corresponding magnitudes are

$$V_1 = [\dot{\vec{R}}^2 + \gamma^2\dot{\vec{L}}^2 - 2\gamma\dot{\vec{R}} \cdot \dot{\vec{L}}]^{1/2} \tag{2.15}$$

$$V_2 = [\dot{\vec{R}}^2 + (1-\gamma)^2\dot{\vec{L}}^2 + 2(1-\gamma)\dot{\vec{R}} \cdot \dot{\vec{L}}]^{1/2} \tag{2.16}$$

$\dot{\vec{R}}$ and $\dot{\vec{L}}$ can be written as

$$\dot{\vec{R}} = \left(\dot{\vec{R}}\right)_{x_o y_o z_o} + \vec{\omega}_o \times \vec{R} \tag{2.17}$$

$$\dot{\vec{L}} = \left(\dot{\vec{L}}\right)_{xyz} + \vec{\omega} \times \vec{L} \tag{2.18}$$

Knowing the system is orbiting in a circular orbit (*i.e.*, $\dot{R} = 0$), and the cable connecting the two spacecraft is moving with a constant speed of $v$, we get

$$\left(\dot{\vec{R}}\right)_{x_o y_o z_o} = 0, \quad \left(\dot{\vec{L}}\right)_{xyz} = \vec{v} \tag{2.19}$$

where $\vec{v}$ is in the direction of $\vec{L}$. Substituting the above relations in Eqs. (2.17)-(2.18), we obtain

$$\dot{\vec{R}} = \vec{\omega}_o \times \vec{R} \tag{2.20}$$

$$\dot{\vec{L}} = \vec{v} + \vec{\omega} \times \vec{L} \tag{2.21}$$

The terms $\dot{\vec{R}}^2$ and $\dot{\vec{L}}^2$ can be written as

$$\dot{\vec{R}}^2 = (\vec{\omega}_o \times \vec{R})^2 \tag{2.22}$$

$$\dot{\vec{L}}^2 = v^2 + (\vec{\omega} \times \vec{L})^2 + 2\vec{v} \cdot (\vec{\omega} \times \vec{L}) \tag{2.23}$$

Writing $\vec{\omega}_o$, $\vec{R}$, $\vec{\omega}$, $\vec{L}$, and $\vec{v}$ in terms of the unit vectors of the respective coordinate frames, we have

$$\omega_o = \dot{\theta}\hat{k}_o, \quad \vec{R} = R\hat{i}_o, \quad \vec{\omega} = (\dot{\theta} + \dot{\beta})\hat{k}, \quad \vec{L} = L\hat{i}, \quad \vec{v} = v\hat{i} \tag{2.24}$$

Inserting these expressions into Eqs. (2.22)-(2.23) and solving, we have

$$\dot{\vec{R}}^2 = \dot{\theta}^2 R^2 \tag{2.25}$$

$$\dot{\vec{L}}^2 = v^2 + (\dot{\theta} + \dot{\beta})^2 L^2 \tag{2.26}$$

Note that as $\vec{v} \perp (\vec{\omega} \times \vec{L})$, $\vec{v} \cdot (\vec{\omega} \times \vec{L}) = 0$.

Next we derive $\dot{\vec{R}} \cdot \dot{\vec{L}}$. Using Eqs. (2.20)-(2.21), we can write

$$\begin{aligned}
\dot{\vec{R}} \cdot \dot{\vec{L}} &= (\vec{\omega}_o \times \vec{R}) \cdot (\vec{v} + \vec{\omega} \times \vec{L}) \\
&= (\vec{\omega}_o \times \vec{R}) \cdot \vec{v} + (\vec{\omega}_o \times \vec{R}) \cdot (\vec{\omega} \times \vec{L}) \\
&= \dot{\theta} R v (\hat{j}_o \cdot \hat{i}) + \dot{\theta}(\dot{\theta} + \dot{\beta}) R L (\hat{j}_o \cdot \hat{j})
\end{aligned} \tag{2.27}$$